DEPARTMENT OF EDUCATION

GRADE 8

MATHEMATICS

STRAND 1

NUMBERS AND APPLICATION

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FOR DEPARTMENT OF EDUCATION
PAPUA NEW GUINEA

GRADE 8

MATHEMATICS

STRAND 1

NUMBERS AND APPLICATION

SUB-STRAND 1:	**FRACTIONS**
SUB-STRAND 2:	**DECIMALS**
SUB-STRAND 3:	**PERCENTAGES**
SUB-STRAND 4:	**RATIOS AND RATES**

Acknowledgements

We acknowledge the contributions of all Secondary and Upper Primary Teachers who in one way or another helped to develop this Course.

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MR. DEMAS TONGOGO
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SECRETARY'S MESSAGE

Achieving a better future by individual students and their families, communities or the nation as a whole, depends on the kind of curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum. The learning outcomes are student-centered with demonstrations and activities that can be assessed.

It maintains the rationale, goals, aims and principles of the national curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision by Flexible, Open and Distance Education as an alternative pathway of formal education.

The course promotes Papua New Guinea values and beliefs which are found in our Constitution and Government Policies. It is developed in line with the National Education Plans and addresses an increase in the number of school leavers as a result of lack of access to secondary and higher educational institutions.

Flexible, Open and Distance Education curriculum is guided by the Department of Education‟s Mission which is fivefold:

- to facilitate and promote the integral development of every individual
- to develop and encourage an education system that satisfies the requirements of Papua New Guinea and its people
- to establish, preserve and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make the education accessible to the poor and physically, mentally and socially handicapped as well as to those who are educationally disadvantaged.

The college is enhanced through this course to provide alternative and comparable pathways for students and adults to complete their education through a one system, two pathways and same outcomes.

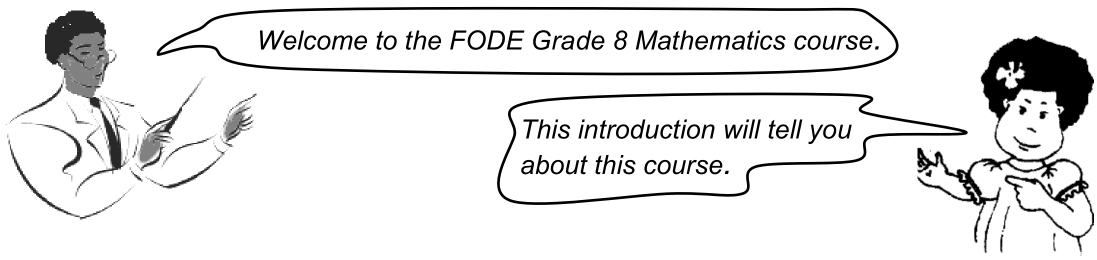
It is our vision that Papua New Guineans‟ harness all appropriate and affordable technologies to pursue this program.

I commend all the teachers, curriculum writers and instructional designers who have contributed towards the development of this course.

....

DR. UKE KOMBRA, PhD
Acting Secretary for Education

COURSE INTRODUCTION



HOW TO STUDY YOUR GRADE 8 MATHEMATICS COURSE?

1. YOUR LESSONS

In Grade 8 Mathematics there are 6 books for you to study. Each book corresponds to each of the six strands of the course.

Strand 1: Number and Application
Strand 2: Space and Shapes
Strand 3: Measurements 1
Strand 4: Measurements 2
Strand 5: Chance and Data
Strand 6: Patterns and Algebra

Each strand is divided into 4 sub-strands and each sub-strand consists of a minimum of 5 to 7 lessons.

Here is a list of the Strands in this Grade 8 course and the Sub-strand you will study:

STRAND	SUB-STRAND	TITLE
1 NUMBER AND APPLICATION	1	FRACTIONS
	2	DECIMALS
	3	PERCENTAGES
	4	RATIOS AND RATES
2 SPACE AND SHAPES	1	ANGLES AND SHAPES
	2	CIRCLES
	3	SIMILARITY AND SCALE DRAWING
	4	TESSELLATION
3 MEASUREMENTS 1	1	LENGTH
	2	AREA
	3	SURFACE AREA
	4	VOLUME AND CAPACITY
4 MEASUREMENTS 2	1	WEIGHTS
	2	TEMPERATURE
	3	TIME
	4	MAPS AND COORDINATES

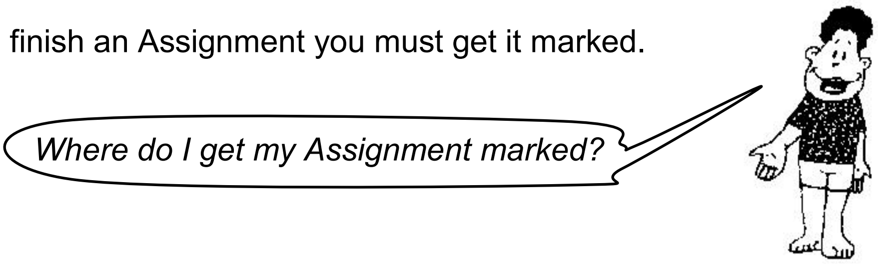
5 CHANCE AND DATA	1	GRAPHS
	2	STATISTICS
	3	SETS
	4	PROBABILITY
6 PATTERNS AND ALGEBRA	1	ALEBRAIC EXPRESSIONS
	2	INDICES
	3	POLYNOMIALS
	4	MATHEMATICAL EQUATIONS

2. YOUR ASSIGNMENTS

In this course you will also do six ASSIGNMENTS.

You will study Strand 1 and do Assignment 1 at the same time. Then you will study Strand 2 and do Assignment 2 at the same time, and so on up to Assignment 6.

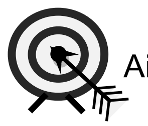
When you finish an Assignment you must get it marked.



1. **Students in a Registered Study Centre** must give their finished Assignments to their Supervisor for marking.
2. **Students who study at Home by themselves but who live in the Provinces** must send their finished Workbooks to their Provincial Centres for marking.

3. LESSON ICONS

Below are the icons used by FODE in its courses

 Lesson Introduction  Aims

 Summary  Activities/Practice Exercises

STRAND 1: NUMBER AND APPLICATION

Dear Student,

This is the first Strand of the Grade 8 Mathematics Course. it is based on the NDOE upper Primary Mathematics Syllabus and Curriculum Framework for Grade 8. It forms part of the continuum of Mathematics learning from Grade 6 to Grade 8, providing a foundation for the grade 9 and grade 10 mathematics courses.

This strand consists of four Sub-strands:

Sub-strand 1: Fractions
Sub-strand 2: Decimals
Sub-strand 3: Percentages
Sub-strand 4: Ratios and Rates

Sub-strand 1 – **Fractions** - In this topic you will extend further your knowledge of fractions and the application of fractions in problem solving in real-life situations.

Sub-strand 2 – **Decimals** – In this topic you will extend further your knowledge of decimals and be able to use decimals to solve real- life problems.

Sub-strand 3 – **Percentages** – In this topic you will extend further your knowledge of percentages and be able to solve problems that involve percentages.

Sub-strand 4 – **Ratios and Rates** – In this topic you will recognize and relate rates to graphs and apply ratios and rates in solving problems in real life.

You will find that each lesson has reading materials to study, worked examples to help you, and a Practice Exercise for you to complete. The answers to the practice exercises are given at the end of each sub-strand.

All lessons are written in simple language with comic characters to guide you and many worked examples to help you.

We hope you enjoy studying this strand.

All the best!

Mathematics Department
FODE

STUDY GUIDE

Follow the steps given below as you work through the Strand.

Step 1: Start with SUB-STRAND 1 Lesson 1 and work through it.

Step 2: When you complete Lesson 1, do Practice Exercise 1.

Step 3: After you have completed Practice Exercise 1, check your work. The answers are given at the end of SUB-STRAND 1.

Step 4: Then, revise Lesson 1 and correct your mistakes, if any.

Step 5: When you have completed all these steps, tick the Lesson check-box on the Contents Page like this:

$\boxed{\surd}$ Lesson 1: Comparing and Ordering Fractions

Then go on to the next Lesson. Repeat the process until you complete all of the lessons in Sub-strand 1.

> As you complete each lesson, tick the check-box for that lesson, on the Contents Page, like this $\boxed{\surd}$ .This helps you to check on your progress.

Step 6: Revise the Sub-strand using Sub-strand 1 Summary, then do Sub-strand Test 1 in Assignment 1.

Then go on to the next Sub-strand. Repeat the process until you complete all of the four Sub-strands in Strand 1.

<u>Assignment:</u> (Four Sub-strand Tests and a Strand Test)
When you have revised each Sub-strand using the Sub-strand Summary, do the Sub-strand Test in your Assignment Book. The Course Book tells you when to do each Sub-strand Test.

When you have completed the four Sub-strand Tests, revise well and do the Strand Test. The Assignment Book tells you when to do the Strand Test.

> The Sub-strand Tests and the Strand Test in the Assignment will be marked by your Distance Teacher. The marks you score in each Assignment Book will count towards your final mark. If you score less than 50%, you will repeat that Assignment Book.

Remember, if you score less than 50% in three Assignments, your enrolment will be cancelled. So, work carefully and make sure that you pass all of the Assignments.

SUB-STRAND 1

FRACTIONS

SUB-STRAND 1: FRACTIONS

Introduction

There are times when most mathematics students wish that the world was made up of whole numbers only. But those non-whole numbers called **fractions** really make the world a wonderful place. In any case, fractions are here to stay and this sub-strand helps you to examine and explore or look into them in all their wondrous workings.

Understanding fractions and where they come from and why they look the way they do helps when you are working with them.

In Grade 7 Mathematics, you learnt everything about fractions, their names, properties and operations. In this Sub-strand, you will revise and further extend the knowledge and skills learnt in applying fractions to problems in real life situations.

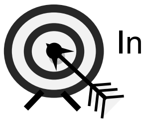
Examples,

1. You bought $6\frac{1}{2}$ kg of sirloin steak and you want to cut it into pieces that weigh $\frac{3}{4}$ of a kg each, how many pieces will there be?
2. You ate two-third kg of a three-fourth kg box of mango. How much mango did you eat?
3. In Mr. Pipi"s will, he gave $\frac{4}{7}$ of his money to the Red Cross and $\frac{1}{3}$ to other charitable institutions. How much was left to his children"s inheritance?

This Sub-strand gets down to the basics with the skills involved in manipulating fractions so you can answer and work out the solutions to different problems.

Lesson 1: Comparing and Ordering Fractions

Welcome to your Grade 8 Mathematics first lesson.



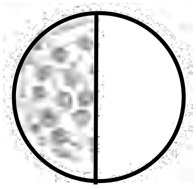
In this lesson, you will:

- describe and revise fractions and their properties.
- compare fractions and order them according to size.

In Grade 7, we learnt that a fraction is a part of the whole and represents a comparison of two whole numbers.

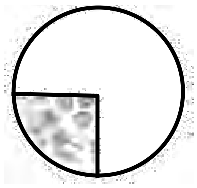
Let‟s revise what we have learnt.

Slice a pizza into parts, and you will have fractions.



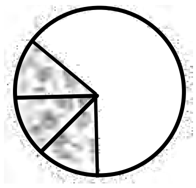
$\frac{1}{2}$

(One-Half)



$\frac{1}{4}$

(One-Quarter)



$\frac{3}{8}$

(Three-Eighths)

The top number tells how many slices you *have* and the bottom number tells how many slices the pizza was *cut into*.

We call the top number the **Numerator**, it is the number of parts you have.

We call the bottom number the **Denominator**. It is the number of parts the whole is divided into.

Thus, in the fraction;

$$\frac{\mathbf{3} \longrightarrow \text{numerator}}{\mathbf{8} \longrightarrow \text{denominator}}$$

Fractions with the same denominators are called **like or similar fractions**. Examples are $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{5}{4}$. On the other hand, **dissimilar fractions or unlike fractions** are those with different denominators such as $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{9}{8}$.

You also learnt in your Grade 7 mathematics the other kinds of fractions.

Let‟s look at them again in case you forgot.

Here are the other kinds of fractions.

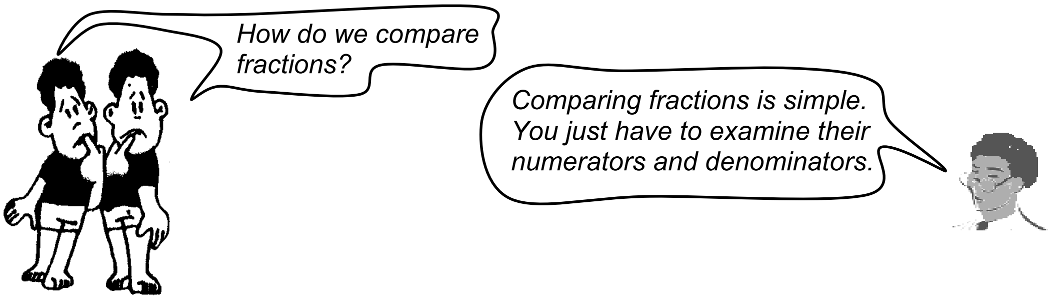
A fraction whose numerator is less than its denominator and whose value is less than 1 whole is called a **proper fraction**.

Examples of Proper Fractions: $\frac{1}{5}, \frac{2}{3}, \frac{3}{4}, \frac{5}{7}$ and $\frac{4}{9}$

A fraction whose numerator is greater than its denominator and whose value is greater than or equal to 1 whole is called an **improper fraction**.

Examples of Improper fractions: $\frac{6}{5}, \frac{5}{3}, \frac{4}{4},$ and $\frac{9}{6}$

Now we are going to compare fractions.



The following are to be remembered when we compare and order fractions.

1. **When comparing similar fractions, the fraction with the greater numerator has the greater value.**

Consider the examples:

Example 1 Compare $\frac{3}{7}$ and $\frac{12}{7}$

Answer: $\frac{3}{7} < \frac{12}{7}$

The first fraction $\frac{3}{7}$ is less than $\frac{12}{7}$. The greater fraction is $\frac{12}{7}$ since it has a greater numerator, 12 than 3 in $\frac{3}{7}$.

Example 2

Arrange the following fractions in ascending order (that is, from the fraction with the least value to the fraction with the highest value):

$$\frac{4}{5}, \frac{8}{5}, \frac{6}{5}, -\frac{2}{5}, \frac{1}{5}$$

Since, $-\frac{2}{5} < \frac{1}{5} < \frac{4}{5} < \frac{6}{5} < \frac{8}{5}$

Answer: $-\frac{2}{5}, \frac{1}{5}, \frac{4}{5}, \frac{6}{5}, \frac{8}{5}$

2. When the numerators of the fractions are the same, the fraction with the smaller denominator has a greater value.

Example 1 Compare $\frac{6}{4}$ and $\frac{6}{12}$

Answer: $\frac{6}{4} > \frac{6}{12}$

The first fraction $\frac{6}{4}$ has a smaller denominator, which is 4, compared to the second fraction $\frac{6}{12}$ which has 12. The first fraction therefore has the greater value because the denominator is smaller.

Example 2 Which of the following fractions has the least value?

$\frac{3}{9}, \frac{3}{2}, \frac{3}{12}, \frac{3}{4}, \frac{3}{7}$

Since, $\frac{3}{2} > \frac{3}{4} > \frac{3}{7} > \frac{3}{9} > \frac{3}{12}$

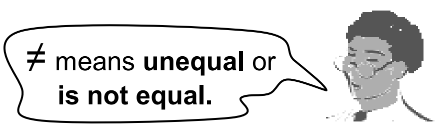
Answer: $\frac{3}{12}$

$\frac{3}{12}$ has the least value because it has the largest denominator.

3. If the product of the numerator of the first fraction and the denominator of the second fraction is greater than the product of the denominator of the first fraction and the numerator of the second fraction, then, the first fraction is greater than the second fraction. In much simpler terms, the *cross multiplication* process is a quicker method of determining which fraction is greater.

See the example below.

Example Compare $\frac{3}{4}$ and $\frac{5}{8}$



Solution: $3 \times 8 = 5 \times 4$

$24 \neq 20$

Therefore, $\frac{3}{4}$ has a greater value than $\frac{5}{8}$. Hence, $\frac{3}{4} > \frac{5}{8}$.

To recap what we have discussed, we have the following.

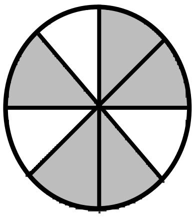
To compare fractions,

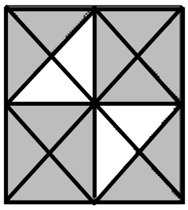
- **when the denominators of the fractions are the same, the fraction with the greater numerator has the greater value.**
- **when the numerators of the fractions are the same, the fraction with the smaller denominator has the greater value.**
- **if the product of the numerator of the first fraction and the denominator of the second fraction is greater than the product of the denominator of the first fraction and the numerator of the second fraction, then, the first fraction is greater than the second fraction. In much simpler terms, the cross multiplication process is the quicker method of determining which fraction is greater or smaller.**

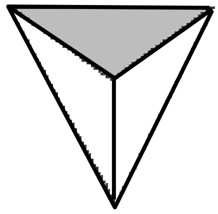
NOW DO PRACTICE EXERCISE 1

Practice Exercise 1

1. What fraction of each diagram is shaded?

(a) 

(b) 

(c) 

2. Write >, < , or = at each blank space provided between the two fractions.

a) $\frac{8}{12}$ ______ $\frac{3}{12}$

b) $\frac{4}{7}$ ______ $\frac{5}{8}$

c) $\frac{9}{6}$ ______ $\frac{9}{4}$

d) $\frac{6}{7}$ ______ $\frac{8}{9}$

e) $\frac{10}{6}$ ______ $\frac{15}{9}$

f) $\frac{4}{5}$ ______ $\frac{3}{4}$

g) $\frac{3}{5}$ ______ $\frac{4}{7}$

h) $\frac{2}{9}$ ______ $\frac{4}{7}$

i) $\frac{8}{9}$ ______ $\frac{7}{8}$

j) $\frac{2}{7}$ ______ $\frac{3}{5}$

3. Which of the following fractions in each set has the least value? Put a circle around your answer.

a) $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$

b) $\frac{1}{9}, \frac{3}{9}, \frac{4}{9}, \frac{6}{9}$

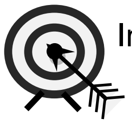
c) $\frac{5}{9}, \frac{7}{10}, \frac{3}{4}$

d) $\frac{3}{4}, \frac{9}{6}, \frac{8}{9}$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 2: Renaming Fractions

You learnt how to compare fractions in the previous lesson.



In this lesson, you will:

- rename fractions
- determine if a fraction is in its simplest form
- rename mixed numbers

Do you remember finding equivalent fractions in your Grade7 Mathematics?

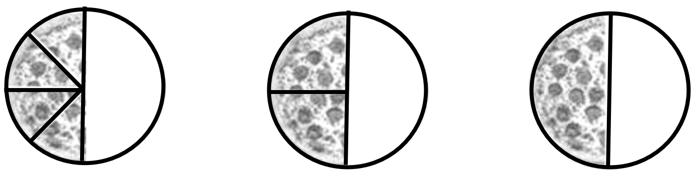
Yes! I still recall doing it but I forget how. Can you explain it again please?

First we will revise the meaning of equivalent fractions.

Equivalent fractions are fractions that may look different but actually have the same value or represent the same part of an object.

Some fractions may look different, but are really the same, for example:

If a pie is cut into two pieces, each piece is one-half of the pie. If a pie is cut into 4 pieces, then two pieces represent the same amount of pie that $\frac{1}{2}$ did. If a pie is cut into 8 pieces, then four pieces represent the same amount of pie that $\frac{2}{4}$ and $\frac{1}{2}$ did. We say that $\frac{1}{2}$ is equivalent to $\frac{2}{4}$ and $\frac{4}{8}$.



$$\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$$

There are two ways to find equivalent fractions.

1. **Building Fractions** - fractions are determined to be equivalent by multiplying the numerator and denominator of one fraction by the same number.

   Example 1 $\quad \frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$

$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

$$\frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$$

$$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$

The fractions $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}$ are equivalent fractions. The simplest is $\frac{1}{2}$.

Hence, we say that $\frac{1}{2}$ is the simplest form.

2. **Simplifying or reducing a fraction**- dividing the numerator and the denominator of a fraction by the same number. This number may either be any common factor or the greatest common factor (GCF) of the numerator and the denominator.

Examples 2

$$\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$ GCF of 6 and 10 is 2.

$$\frac{9}{15} = \frac{9 \div 3}{15 \div 3} = \frac{3}{5}$$ GCF of 9 and 15 is 3

$$\frac{12}{20} = \frac{12 \div 4}{20 \div 4} = \frac{3}{5}$$ GCF of 12 and 20 is 4

$$\frac{15}{25} = \frac{15 \div 5}{25 \div 5} = \frac{3}{5}$$ GCF of 15 and 25 is 5

The fractions $\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \frac{15}{25}$ are equivalent fractions. The simplest is $\frac{3}{5}$.

Hence, we say, $\frac{3}{5}$ is the simplest form.

With the discussion and examples given so far, we are led to the following generalizations.

- **Given a fraction, equivalent fractions can be obtained by multiplying or dividing the numerator and denominator of the given fraction by the same number.**
- **A fraction is in its simplest form when the numerator and denominator have no common factor except 1.**

To compare fractions with different denominators, change these fractions so that they have the same denominator, then compare the numerators.

The following examples found on the next page show how to determine whether fractions are equivalent.

Here are some examples.

Example 3 Is $\frac{6}{8}$ equivalent to $\frac{9}{12}$?

Solution: $\frac{6}{8} = \frac{2 \times 3}{2 \times 4} = \frac{3}{4}$

$\frac{9}{12} = \frac{3 \times 3}{3 \times 4} = \frac{3}{4}$

Therefore, $\frac{6}{8} = \frac{9}{12}$

Example 4 Is $\frac{4}{5}$ and $\frac{7}{10}$ equivalent?

Solution: $\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10}$

$\frac{8}{10}$ is greater than $\frac{7}{10}$

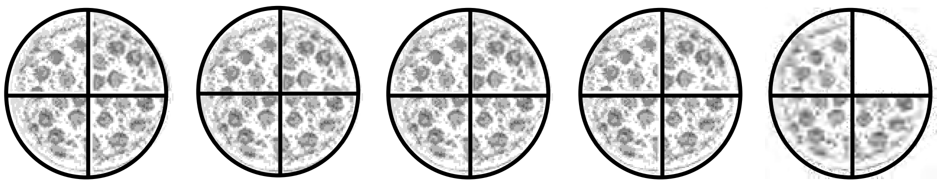
That is $\frac{4}{5} > \frac{7}{10}$, Therefore, the two fractions are **not** equivalent.

Other kind of fractions can also be renamed.

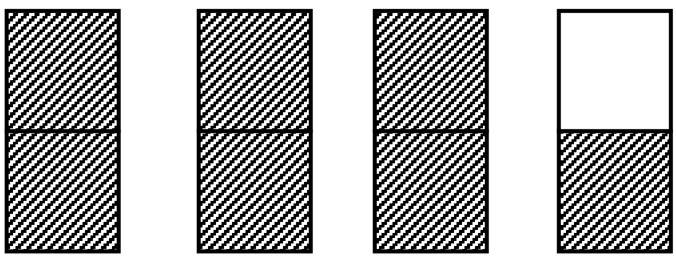
An improper fraction can also be expressed as a **mixed number** - a combination of a whole number and a fraction.

Consider the figure below. Count the number of wholes. Take note of the number of equal parts in each whole and the number of shaded parts.

a)



b)



Notice the following observations:

Figure a shows $\frac{19}{4}$ since there are 19 quarters shaded. But how many wholes are shaded? The fraction can be written as $4\frac{3}{4}$, read as " four and three fourths."

Figure b shows $\frac{7}{2}$. How many halves are shaded? How many wholes are there? This can be written as $3\frac{1}{2}$, read as "three and one half."

The numerals $4\frac{3}{4}$ and $3\frac{1}{2}$ are the mixed number forms for $\frac{19}{4}$ and $\frac{7}{2}$ respectively.

Every improper fraction can be expressed as a mixed number and vice versa.

We have learned that $\frac{19}{4}$ is an improper fraction. If we simplify the improper fraction, the result is the mixed number as illustrated below.

$$\frac{19}{4} \;=\; 4\,\begin{array}{r} 4 \\ \hline \big)\,19 \\ \underline{16} \\ 3 \end{array} \;=\; 4\frac{3}{4}$$

However, to reverse the process, that is to convert the mixed number $4\frac{3}{4}$ to improper fraction we have $4\frac{3}{4} = \frac{(4 \text{ x } 4)+3}{4} = \frac{16+3}{4} = \frac{19}{4}$

Hence, $4\frac{3}{4} = \frac{19}{4}$

Remember:

- **To change a mixed number into an improper fraction, multiply the whole number by the denominator and add it to the numerator of the fractional part.**
- **To change an improper fraction into a mixed number, divide the numerator by the denominator. The remainder is the numerator of the fractional part.**

NOW DO PRACTICE EXERCISE 2

Practice Exercise 2

1. Following the pattern, add two equivalent fractions to the list.

a) $\frac{1}{5}, \frac{2}{10}, ..., ..., \frac{5.}{25}$

b) $\frac{2}{9}, \frac{4}{18}, \ldots, \ldots, \frac{10}{45}$

c) $\frac{3}{4}, \frac{9}{12}, \frac{15}{20}, \ldots, \ldots$

d) $\frac{4}{11}, \frac{12}{33}, \frac{20}{55}, \ldots, \ldots$

e) $\frac{1}{3}, \frac{4}{12}, \frac{5}{15}, \ldots, \ldots$

2. Change the following fractions to simplest form.

a) $\frac{12}{60}$

b) $\frac{16}{28}$

c) $\frac{9}{60}$

d) $\frac{22}{40}$

e) $\frac{12}{30}$

f) $\frac{56}{64}$

g) $\frac{18}{50}$

h) $\frac{24}{48}$

i) $\frac{16}{32}$

j) $\frac{35}{60}$

3. Change the following improper fractions to mixed numbers.

a) $\frac{8}{3}$

b) $\frac{7}{2}$

c) $\frac{9}{5}$

d) $\frac{36}{28}$

e) $\frac{38}{19}$

f) $\frac{9}{4}$

g) $\frac{27}{8}$

h) $\frac{16}{7}$

i) $\frac{31}{7}$

j) $\frac{21}{5}$

4. Convert the following mixed numbers to improper fractions.

a) $9\frac{3}{11}$

b) $5\frac{3}{4}$

c) $3\frac{4}{7}$

d) $4\frac{1}{3}$

e) $1\frac{1}{3}$

f) $10\frac{1}{2}$

g) $3\frac{7}{8}$

h) $5\frac{1}{7}$

i) $4\frac{3}{8}$

j) $2\frac{2}{5}$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 3: Addition and Subtraction of Fractions and Mixed Numbers

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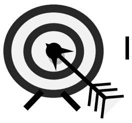
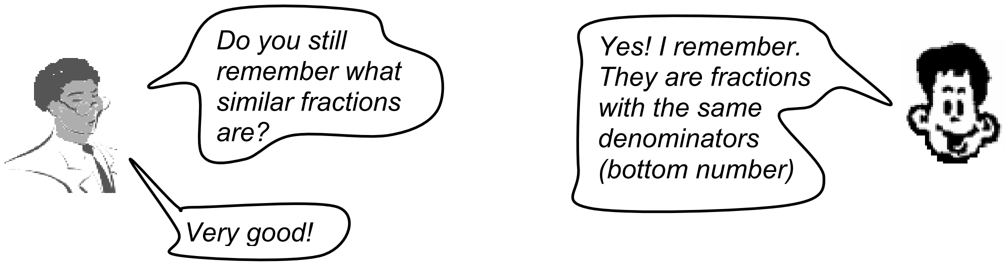
You learnt how to rename fractions and mixed numbers in the previous lesson.



In this lesson, you will:

- define the Least Common Denominator (LCD)
- add and subtract fractions and mixed numbers
- solve real life problem situations involving addition and subtraction of fractions and mixed numbers.

In adding or subtracting fractions, there is one important thing to remember; we can only add or subtract similar fractions.



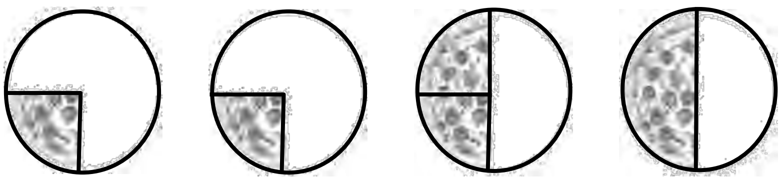
Here are some examples on the adding and subtracting of fractions.

Adding and subtracting similar fractions

You can add or subtract fractions easily if the bottom number (the *denominator*) is the same.

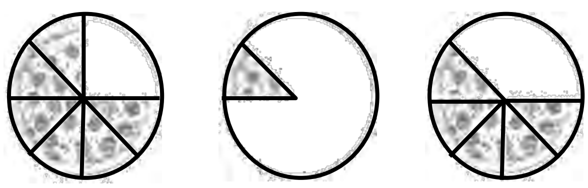
Example 1

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$



Example 2

$$\frac{6}{8} - \frac{1}{8} = \frac{5}{8}$$

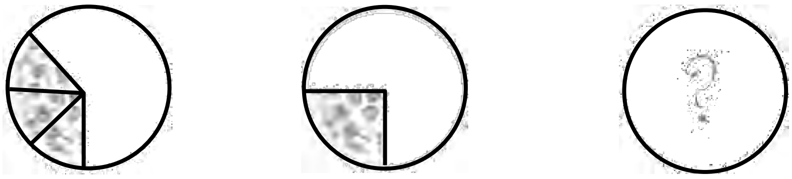


Adding and Subtracting Fractions with Different Denominators

But what if the **denominators** are not the same? As in this example:

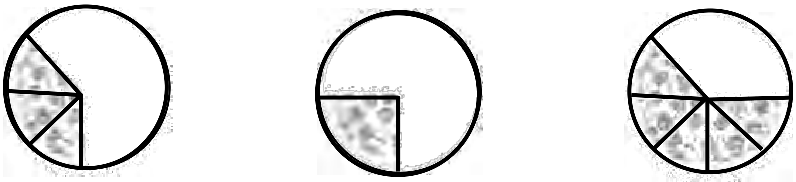
Example 3

$$\frac{3}{8} \quad + \quad \frac{1}{4} \quad = \quad ?$$



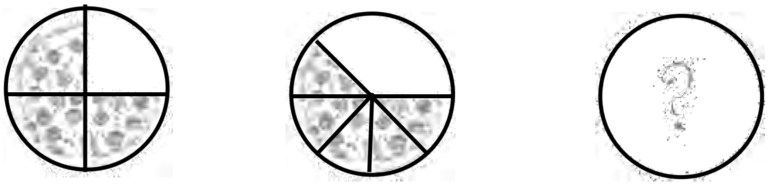
You must make the denominators the same. In this case it is easy, because we know that $\frac{1}{4}$ is the same as $\frac{2}{8}$:

$$\frac{3}{8} \quad + \quad \frac{2}{8} \quad = \quad \frac{5}{8}$$



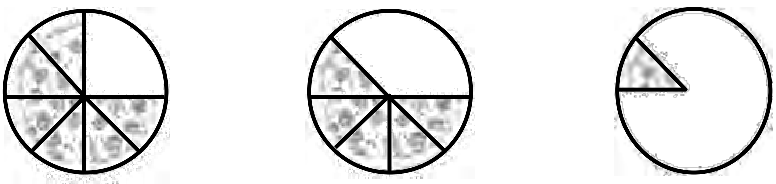
Example 4

$$\frac{3}{4} \quad - \quad \frac{5}{8} \quad = \quad ?$$



Make the denominators the same. In this case it is easy, because we know that $\frac{3}{4}$ is the same as $\frac{6}{8}$

$$\frac{6}{8} \quad - \quad \frac{5}{8} \quad = \quad \frac{1}{8}$$



In these examples, it was easy to make the denominators the same, but it can be harder... so we may need to use least common denominator (LCD) to make them the same.

The Least Common Denominator (LCD) is the smallest number that all the given denominators will divide into evenly. It is the Least Common Multiple (LCM) of two or more given denominators.

Here is how to find the Least Common Denominator:

- Find the Greatest Common Factor (GCF) of the denominators.
- Multiply the denominators together.
- Divide the product of the denominators by the Greatest Common Factor (GCF).

Here is an example.

Example Find the LCD of $\frac{2}{9}$ and $\frac{3}{12}$.

Solution: Steps:

1. Determine the GCF of 9 and 12 which is 3.

2. Either you multiply the denominators and divide by the GCF.

   $(9 \times 12 = 108 \quad , \quad \frac{108}{3} = 36)$

   OR Divide one of the denominators by the GCF and multiply the quotient times the other denominator

   $(\frac{9}{3} = 3 \ , \ 3 \times 12 = 36)$

Hence, the LCD of $\frac{2}{9}$ and $\frac{3}{12}$ is 36.

How to rename fractions and use the Least Common Denominator:

- Divide the LCD by one denominator.
- Multiply the numerator times this quotient.
- Repeat the process for the other fraction(s)

Example: Add $\frac{2}{9}$ and $\frac{3}{12}$ using the LCD which is 36.

Solution:

Steps

1. First fraction $\frac{2}{9}$: $\frac{36}{9}$ = 4, 2 x 4 = 8, first fraction is renamed as $\frac{8}{36}$
2. Second fraction $\frac{3}{12}$: $\frac{36}{12}$ = 3, 3 x 3 = 9, second fraction is renamed as $\frac{9}{36}$

It is now possible to add $\frac{2}{9} + \frac{3}{12}$ since $\frac{8}{36} + \frac{9}{36} = \frac{17}{36}$

REMEMBER
- **To add similar fractions, add the numerators and simply copy the denominator.**
- **To add dissimilar fractions or fractions with different denominators, convert the fractions to equivalent fractions with the same denominators by finding the Least Common Denominator (LCD).**

Here are some examples showing the addition and subtraction of fractions and mixed numbers.

1. $9\frac{1}{4} + \frac{5}{7} = 9 + \frac{1}{4} + \frac{5}{7}$ renaming $9\frac{1}{4}$ to $9 + \frac{1}{4}$

$= 9 + \left(\frac{1}{4} + \frac{5}{7}\right)$ associative property of addition

$= 9 + \left(\frac{7}{28} + \frac{20}{28}\right)$ finding the LCD

$= 9 + \frac{27}{28}$ adding the numerators

$= 9\frac{27}{28}$ answer

The subtraction of fractions follow the same rules, except you subtract the whole numbers and subtract the numerators.

In a situation when subtraction requires borrowing, rename the minuend (the top or first mixed number) so that it is one smaller in the whole number part and the equivalent of one more in the fraction part.

Example: $8\frac{2}{5} = 7 + \frac{5}{5} + \frac{2}{5} = 7 + \frac{7}{5} = 7\frac{7}{5}$

Once this has been completed proceed with other subtraction problems. Remember, an LCD is still needed and should be found before you borrow or rename.

To illustrate subtraction of mixed numbers, consider the examples below.

1. $8\frac{1}{2} - 2\frac{1}{3} = 8\frac{3}{6} - 2\frac{2}{6}$ renaming the fractions with LCD (6 is the LCD)

$= (8 - 2) + \left(\frac{3}{6} - \frac{2}{6}\right)$ associative property

$= 6 + \frac{1}{6}$ subtract whole number parts and fractional parts

$= 6\frac{1}{6}$ **Answer**

2. $12\frac{1}{4} - 5\frac{2}{3} = 12\frac{3}{12} - 5\frac{8}{12}$ renaming the fractions with LCD

$= 11 + \frac{12}{12} + \frac{3}{12} - 5\frac{8}{12}$ borrow one from 12

$= 11\frac{15}{12} - 5\frac{8}{12}$ rename $12\frac{3}{12}$ to $11\frac{15}{12}$

$= 11\frac{15}{12} - 5\frac{8}{12}$ rename $12\frac{3}{12}$ to $11\frac{15}{12}$

$= (11 - 5) + \left(\frac{15}{12} - \frac{8}{12}\right)$ associative property

$= 6 + \frac{7}{12}$ subtract whole number parts and fractional parts

$= 6\frac{7}{12}$ **Answer**

Now, let‟s look at and solve example problems in real life situations involving addition and subtraction of decimals.

See next page.

Example 1

Melrose promised her parents to practice the violin for 7 hours this week. If she has practiced $4\frac{3}{4}$ hours so far this week, how many more hours does Melrose need to practice?

Solution: Subtraction must be used to find the difference between the hours in which she still has to practice and the hours in which she has actually practiced.

so, $7 - 4\frac{3}{4} = 6\frac{4}{4} - 4\frac{3}{4}$ rename 7 to fraction with the same denominator of 4.

$= 2\frac{1}{4}$ hours **Answer**

Example 2

Oala cut two pieces of fabric from a bolt of fabric containing 30 metres. The pieces she cut were $2\frac{3}{4}$ metres and $3\frac{2}{3}$ metres. How much fabric remains in the bolt?

Solution: First find the total length Oala cut off from the bolt by adding together the two desired lengths.

Total amount $= 2\frac{3}{4} + 3\frac{2}{3}$

$= 2\frac{9}{12} + 3\frac{8}{12}$

$= 5\frac{17}{12}$

$= 6\frac{5}{12}$ metres

Then subtract the sum from the amount of material that was initially on the bolt to find out how much is left.

Amount left $= 30 - 6\frac{5}{12}$

$= 29\frac{12}{12} - 6\frac{5}{12}$

$= 23\frac{7}{12}$ metres

Therefore, $23\frac{7}{12}$ metres of fabric remain in the bolt.

NOW DO PRACTICE EXERCISE 3

Practice Exercise 3

1. Do the indicated operations. Express your answers in simplest form.

   a) $\frac{3}{4}+\frac{2}{5}$

   b) $\frac{2}{7}-\frac{1}{6}$

   c) $8 + 6\frac{1}{2}$

   d) $6\frac{7}{9} - 2$

   e) $12\frac{2}{9} + 2$

   f) $16\frac{1}{3} + 8\frac{2}{9}$

   g) $16\frac{1}{3} - 9\frac{1}{5}$

   h) $\frac{3}{9} + \frac{5}{9} + \frac{1}{9}$

---

2. Solve the following problems.

   a) Pies were delivered to a party. Each pie was cut up into the same number of pieces. After the party, parts of four different pies were left uneaten.

   The fractions that remain were: $\frac{5}{8}, \frac{1}{8}, \frac{3}{4}, \frac{1}{2}$

   How many pies is the sum of the fractions equivalent to?

   b) A bottle of soy sauce is three-fifths full. A recipe requires half a bottle of soy sauce.

   What fraction will be left in the bottle after the recipe is done?

3. Katu has $8\frac{2}{7}$ metres of material. She uses $5\frac{1}{2}$ metres to make a table cloth.

   How much material is left?

---

4. Casey spent $\frac{1}{8}$ of her pocket money going to the gym and $\frac{1}{5}$ to buy a book. She put the rest in the bank.

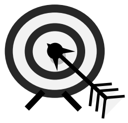
   a) What fraction of the money did Casey put in the bank?

   b) If Casey had K160 pocket money, how much did she spend on each item?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 4: Multiplication of Fractions and Mixed Numbers

You learnt how to add and subtract fractions and mixed numbers in the previous lesson.



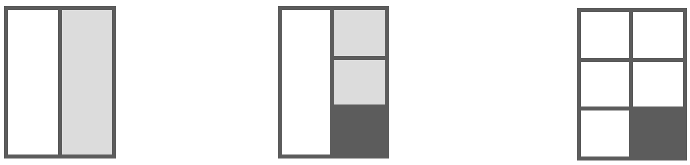
In this lesson, you will

- multiply fractions and mixed numbers
- solve problems in real life situations involving multiplication of fractions and mixed numbers

You studied multiplication of fractions in Grade 7. Here you will study and extend your knowledge and understanding of how to multiply fractions and mixed numbers.

Look at the following diagrams.

a)



$\frac{1}{2}$ of the region is shaded

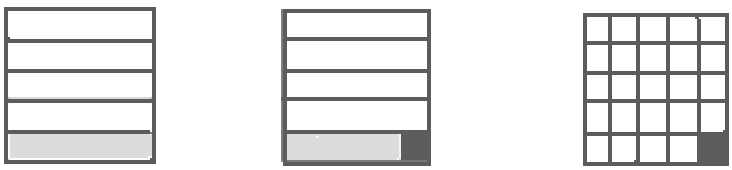
$\frac{1}{3}$ of $\frac{1}{2}$ is darkened

$\frac{1}{6}$ of the region is darkened

Multiplication Facts

$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

b)



$\frac{1}{5}$ of the region is shaded

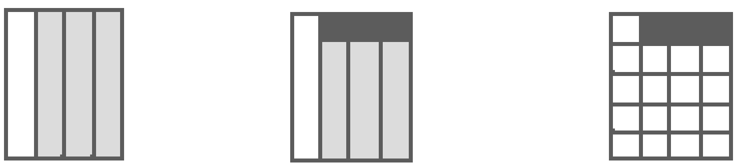
$\frac{1}{5}$ of $\frac{1}{5}$ is darkened

$\frac{1}{25}$ of the region is darkened

Multiplication Facts

$\frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$

c)



$\frac{3}{4}$ of the region is shaded

$\frac{1}{5}$ of $\frac{3}{4}$ is darkened

$\frac{3}{20}$ of the region is darkened

Multiplication Facts

$\frac{1}{5} \times \frac{3}{4} = \frac{3}{20}$

Let us rewrite all the preceding multiplication statements suggested by the diagrams and see if we can state the rule for multiplying fractions.

1. $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

2. $\frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$

3. $\frac{1}{5} \times \frac{3}{4} = \frac{3}{20}$

Look at the numerators of the products and the numerators of the fractional factors. What can you say about the product of the numerators of the factors? How then do we find the numerator of a product?

Now, look at the denominators of the products and the denominators of the fractional factors. How do we find the denominators of the product as shown by the examples? How then do we find the product of two fractions?

To find the product of two fractions, multiply the numerators to get the numerator of the product and then multiply the denominators to get the denominator of the product. Simplify the answer if possible.

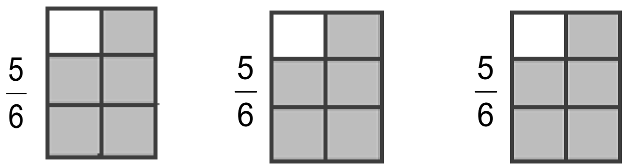
Next, we want to find out how much is 3 five sixths. The following show how we can interpret this problem.

1. $3 \times \frac{5}{6} = \frac{3}{1} \times \frac{5}{6} = \frac{15}{6} = 2\frac{3}{6}$ or $2\frac{1}{2}$

(Multiplication is repeated addition)

2. $3 \times \frac{5}{6} = \frac{5}{6} + \frac{5}{6} + \frac{5}{6} = \frac{15}{6} = 2\frac{3}{6}$ or $2\frac{1}{2}$

3.



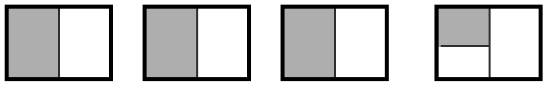
Since a whole number can be written as a fraction, we can adopt the procedure described earlier to find the product of a whole number and a fraction. For example suppose we want to find $15 \times \frac{3}{4}$. Rewriting this, we get

$$15 \times \frac{3}{4} = \frac{15}{1} \times \frac{3}{4} = \frac{45}{4} = 11\frac{1}{4}$$

Thus, we say

To multiply a whole number by a fraction, write the whole number as a fraction and find the product. Simplify the answer if possible.

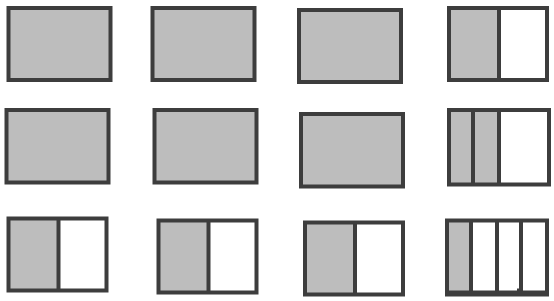
Next, find the product of $\frac{1}{2}$ x $3\frac{1}{2}$. To do this, we use the following diagrams and interpretation.

1. 

2. $\frac{1}{2}$ x $3\frac{1}{2}$ = $\frac{1}{2}$ x $\frac{7}{2}$ = $\frac{7}{4}$ = $1\frac{3}{4}$

Clearly, the same principle for multiplication is used.

Next, find the product of $2\frac{1}{2}$ x $3\frac{1}{2}$. To do this, we consider the following interpretation.

1. 

2. $2\frac{1}{2}$ x $3\frac{1}{2}$. = $\frac{5}{2}$ x $\frac{7}{2}$ = $\frac{35}{4}$ = $8\frac{3}{4}$

Based on the above illustration, how do we find the product of a fraction and a mixed number?

Notice that multiplication of a fraction by a mixed number and multiplication of two mixed fractions follow the same principle for multiplying any two fractions.

- **In multiplying a fraction and a mixed number convert the mixed number to an improper fraction before multiplying.**
- **If both factors are mixed numbers, convert each mixed number to an improper fraction before multiplying.**

Here are some more examples of multiplying fractions and mixed numbers.

1. $\frac{3}{4}$ x $\frac{8}{9}$ = $\frac{24}{36}$ = $\frac{2}{3}$

2. $\frac{3}{4}$ x $\frac{3}{5}$ = $\frac{9}{20}$

3. 4 x $\frac{3}{8}$ = $\frac{12}{8}$ = $1\frac{4}{8}$ or $1\frac{1}{2}$

4. $1\frac{7}{8} \times 2\frac{1}{2} = \frac{15}{8} \times \frac{5}{2} = \frac{75}{16} = 4\frac{11}{16}$

Now, we are going to work out problems in real life situation.

Example 1

Margaret takes $\frac{3}{4}$ of an hour to do her mathematics project. It takes Louisa $2\frac{1}{4}$ times longer than Margaret to do the project. How long does it take Louisa?

Solution: Louisa takes $2\frac{1}{4}$ times longer than Margaret who takes $\frac{3}{4}$ of an hour.

$$\text{so,} \quad 2\frac{1}{4} \times \frac{3}{4} = \frac{9}{4} \times \frac{3}{4}$$

$$= \frac{27}{16}$$

$$= 1\frac{11}{16} \text{ hours}$$

Therefore, Louisa takes $1\frac{11}{16}$ hours to do the project.

Example 2

Two friends in a car travel at 85 kilometres per hour. How far will they go in $2\frac{5}{6}$ hours?

Solution: If the two friends travel 85 km in an hour, then

$$2\frac{5}{6} \times 85 = \frac{17}{6} \times 85$$

$$= \frac{1445}{6}$$

$$= 240\frac{5}{6} \text{ km}$$

Therefore, the two friends will travel a distance of $240\frac{5}{6}$ km in $2\frac{5}{6}$ hours.

NOW DO PRACTICE EXERCISE 4

Practice Exercise 4

1. Multiply the following:

a) $4 \times \frac{1}{5}$

b) $10 \times \frac{1}{7}$

c) $5 \times \frac{5}{3}$

d) $\frac{5}{6} \times \frac{4}{7}$

e) $\frac{7}{8} \times \frac{1}{8}$

f) $2\frac{1}{2} \times 4$

g) $1\frac{1}{3} \times 9$

h) $2\frac{1}{3} \times 2\frac{2}{7}$

i) $\frac{2}{5} \times \frac{2}{3} \times \frac{1}{4}$

j) $2\frac{2}{3} \times 4\frac{1}{2} \times 1\frac{4}{5}$

2. Problem Solving

a) Kopi"s family consumes $5\frac{1}{2}$ kg of rice in a week. How many kilogram of rice will his family consume in $3\frac{1}{2}$ weeks?

Answer:

b) Workers pay $\frac{8}{25}$ of their wages in tax. If a worker"s wage is K520, how much tax does he pay?

Answer:

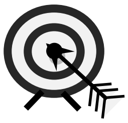
c) Asi collected 5 bags of kaukau in her garden. Each bag holds $3\frac{2}{5}$ kg. What is the total weight of the kaukau?

Answer:

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

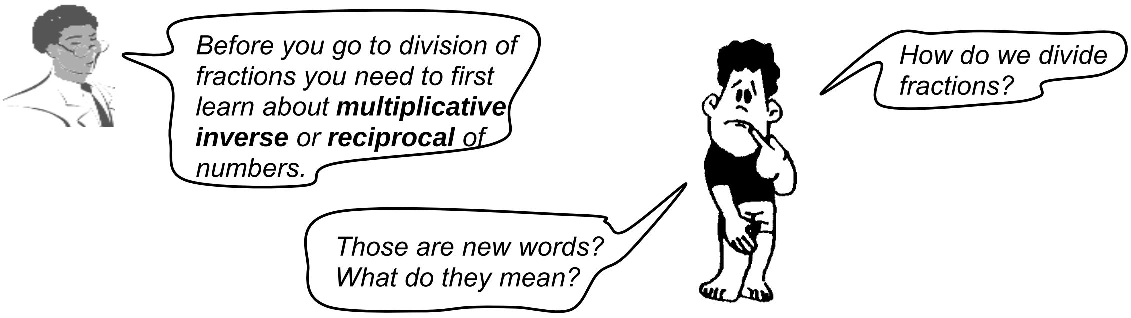
Lesson 5: Division of Fractions

You learnt how to multiply fractions and mixed numbers in the previous lesson.



In this lesson, you will:

- define what the reciprocal of a number is
- find the reciprocal of a number
- divide fractions and mixed numbers
- solve problems in real life situations involving division of fractions and mixed numbers



When the product of two numbers is equal to 1, each of the numbers is called a reciprocal or the multiplicative inverse of the other.

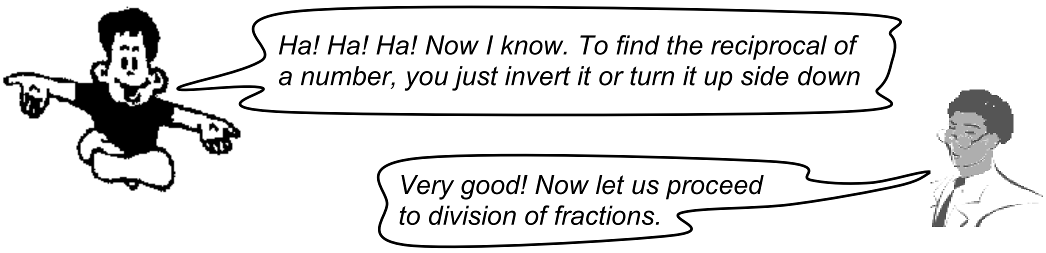
The method is explained as follows:

1) The numbers 3 and $\frac{1}{3}$ are reciprocals of each other because

$$3 \times \frac{1}{3} = \frac{3}{3} \text{ or } 1 \text{ and } \frac{1}{3} \times 3 = \frac{3}{3} \text{ or } 1$$

2) Likewise, $\frac{4}{5}$ is the multiplicative inverse or reciprocal of $\frac{5}{4}$ because

$$\frac{4}{5} \times \frac{5}{4} = \frac{20}{20} \text{ or } 1.$$



The simplest way to divide fractions is to multiply the dividend (first number) by the reciprocal (inverted form) of the divisor (second number). Then write the answer to its simplest form.

It sounds difficult but it is quite simple. The method may be explained as follows:

Dividend ÷ fraction = Dividend x reciprocal of the fraction

Here is an example.

Example 1

Divide: 10 by $\frac{2}{3}$ = $\boxed{}$

Multiply 10 (dividend) by the reciprocal (inverted form) of $\frac{2}{3}$ (divisor).

So, $10 \div \frac{2}{3} = 10 \times \frac{3}{2} \longrightarrow$ (reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$)

$= \frac{10 \times 3}{1 \times 2} \longrightarrow \left[\frac{\text{multiply numerators}}{\text{multiply denominators}}\right]$

$= \frac{30}{2} \longrightarrow$ (simplify)

$= 15 \longrightarrow$ **Answer**

Example 2

Divide: $\frac{5}{6}$ by $\frac{3}{4}$ = $\boxed{}$

$\frac{5}{6} \div \frac{3}{4} = \frac{5}{6} \times \frac{4}{3} \longrightarrow$ (reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$)

$= \frac{5 \times 4}{6 \times 3} \longrightarrow \left[\frac{\text{multiply numerators}}{\text{multiply denominators}}\right]$

$= \frac{20}{18} \longrightarrow$ (Simplify)

$= \frac{10}{9} \longrightarrow$ (Change to mixed number)

$= 1\frac{1}{9} \longrightarrow$ **Answer**

Now, let"s look at the next example.

Example 3

Divide: $2\frac{3}{4}$ by $\frac{1}{2}$ = $\square$

$2\frac{3}{4} \div \frac{1}{2} = \frac{11}{4} \div \frac{1}{2}$ ⟶ change $2\frac{3}{4}$ to improper fraction

$= \frac{11}{4} \times \frac{2}{1}$ ⟶ multiply by the reciprocal of $\frac{1}{2}$ which is $\frac{2}{1}$

$= \frac{22}{4}$ ⟶ $\left[\frac{11 \times 2}{4 \times 1} = \frac{22}{4}\right]$

$= 5\frac{2}{4}$ ⟶ Simplify

$= 5\frac{1}{2}$ ⟶ **Answer**

Example 4

Divide: $5\frac{1}{3} \div 1\frac{3}{5}$ = $\square$

$5\frac{1}{3} \div 1\frac{3}{5} = \frac{16}{3} \div \frac{8}{5}$ ⟶ change $5\frac{1}{3}$ and $1\frac{3}{5}$ to improper fractions

$= \frac{16}{3} \times \frac{5}{8}$ ⟶ multiply by the reciprocal of $\frac{8}{5}$ which is $\frac{5}{8}$.

$= \frac{80}{24}$ ⟶ $\left[\frac{16 \times 5}{3 \times 8} = \frac{80}{24}\right]$

$= \frac{15}{3}$ ⟶ Simplify

$= 5$ ⟶ **Answer**

From all the examples so far you can make the following generalizations.

To divide fractions and mixed numbers,

- **first change all the mixed numbers to improper fractions**
- **change the division sign to a multiplication sign and invert the divisor(reciprocal)**
- **proceed to multiply. Cancel down whenever possible and change any answers that are improper fractions back to mixed numbers. Simplify the answer if possible.**

You know how to divide fractions and mixed numbers, now you are going to apply the knowledge and skills you have learned to solve real life problems involving division of fractions and mixed numbers. See the examples on the next page.

Example 1

The Vinluan family consumes $2\frac{1}{4}$ kg of rice a day. How long will a $51\frac{3}{4}$ kg sack of rice last the family?

Solution: The problem is asking and requiring you to find the number of days in which the Vinluan family will consume all $51\frac{3}{4}$ kg of rice, if they consume $2\frac{1}{4}$ kg of rice each day.

So what we do is to divide $51\frac{3}{4}$ kg by $2\frac{1}{4}$ kg to find the number of days.

so, $51\frac{3}{4}$ kg ÷ $2\frac{1}{4}$ kg $= \frac{207}{4} \div \frac{9}{4} \longrightarrow$ change $51\frac{3}{4}$ and $2\frac{1}{4}$ to improper fractions

$= \frac{207}{4} \times \frac{4}{9} \longrightarrow$ multiply by the reciprocal of $\frac{9}{4}$ is $\frac{4}{9}$

$= \frac{828}{36} \longrightarrow \left[\frac{207 \times 4}{4 \times 9} = \frac{828}{36}\right]$ or by cancellation

$= \frac{207}{9} \longrightarrow$ (simplify)

$= 23$ days $\longrightarrow$ **Answer**

Therefore, $51\frac{3}{4}$ kg will last the family 23 days.

Example 2

Louisa has $2\frac{1}{2}$ cakes to divide between her 8 friends. What fraction does each of her friends receive?

Solution: $2\frac{1}{2} \div 8 = \frac{5}{2} \div 8 \longrightarrow$ change $2\frac{1}{2}$ to improper fraction

$= \frac{5}{2} \times \frac{1}{8} \longrightarrow$ multiply by the reciprocal of 8 which is $\frac{1}{8}$

$= \frac{5}{16} \longrightarrow$ answer

Therefore, each of Louisa"s friends will receive $\frac{5}{16}$ of the cake.

Example 3

The Agao family has a water tank with $40\frac{1}{2}$ kilolitres of water in it. If they use $2\frac{2}{5}$ kilolitres of water per week, how long will it take to empty the tank if it does not rain?

Solution:

$$40\frac{1}{2} \div 2\frac{2}{5} = \frac{81}{2} \div \frac{12}{5} \longrightarrow \text{change to improper fractions}$$

$$= \frac{81}{2} \times \frac{5}{12} \longrightarrow \text{multiply by the reciprocal of } \frac{12}{5} \text{ is } \frac{5}{12}$$

$$= \frac{405}{24} \longrightarrow \left[\frac{81 \times 5}{2 \times 12}\right]$$

$$= 16\frac{21}{24} \longrightarrow \text{simplify}$$

$$= 16\frac{7}{8} \text{ weeks} \longrightarrow \textbf{Answer}$$

NOW DO PRACTICE EXERCISE 5

Practice Exercise 5

1. Write the reciprocal next to each of the following.

a) $\frac{2}{3}$	f) $3\frac{2}{3}$
b) $\frac{1}{5}$	g) 2
c) $\frac{2}{7}$	h) $5\frac{1}{4}$
d) $\frac{7}{8}$	i) $\frac{9}{5}$
e) $\frac{1}{3}$	j) $\frac{5}{9}$

2, Divide the following fractions.

(1) $\frac{3}{7} \div \frac{2}{21} =$	(2) $9 \div \frac{3}{5} =$
(3) $\frac{4}{5} \div \frac{7}{12} =$	(4) $\frac{1}{4} \div \frac{2}{8} =$
(5) $\frac{3}{2} \div \frac{21}{4} =$	(6) $\frac{10}{11} \div 5 =$
(7) $5\frac{1}{4} \div 2\frac{4}{5} =$	(8) $1\frac{1}{4} \div 3\frac{1}{2} =$

3. Solve the following problems.

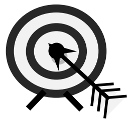
(a) A construction company buys $16\frac{2}{5}$ hectares of land which is to be subdivided into $\frac{2}{5}$ hectare apartment blocks. How many blocks will there be?

(b) An orchard worker packs mangoes into $1\frac{1}{5}$ kilogram bags. If a box contains $10\frac{3}{4}$ kilograms of mangoes, how many bags can be packed from a box of mangoes?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OFSUB-STRAND 1

Lesson 6: Mixed Problems involving Fractions and Mixed Numbers

You learnt how to add, subtract multiply and divide fractions and mixed numbers in the previous lessons.



In this lesson, you will:

- solve problems involving fractions and mixed numbers

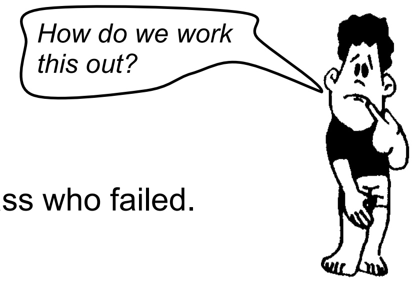
There are mixtures of problems in real life situations involving addition, subtraction, multiplication and division of fractions.

You are now going to look at some of these problems and try to work them out using the mathematical operations and skills you have built up on fractions.

Here are some examples:

Example 1

In a Mathematics class of 60 students, $\frac{1}{3}$ of the class scored very high, $\frac{1}{4}$ scored high and $\frac{1}{6}$ scored fairly high. The others failed.



What part of the class failed?

Solution: The problem asked for the part of the class who failed.

First, you find the following:

Part of the class who scored very high = $\frac{1}{3}$ of 60 = 20

Part of the class who scored high = $\frac{1}{4}$ of 60 = 15

Part of the class who scored fairly high = $\frac{1}{6}$ of 10 = 10

To find what part of the class failed, you have

$$\begin{aligned} \text{Part of class failed} &= 60 - (20 + 15 + 10) \\ &= 60 - 45 \\ &= 15 \qquad \textbf{Answer} \end{aligned}$$

15 is $\frac{1}{4}$ of 60

Therefore, $\frac{1}{4}$ of the class failed.

Example 2

In a mathematics class of 54 students, there are 30 girls. If two thirds of the girls and half the boys are members of the Mathematics Club, then how many students in the class are members of the Club?

The problem is asking how many of the students in the class are members of the Club.

Solution: First you have to find the number of boys in the class

Number of boys = number of students in the class – number of girls

Number of boys = 54 – 30 = 24

To find the number of Club members, you have:

Number of Club members = $\frac{2}{3}$ of 30 and $\frac{1}{2}$ of 24

$= \frac{2}{3} \times 30 + \frac{1}{2} \times 24$

$= 20 + 12$

$= 32$ **Answer**

Therefore, there are 32 students in the class who are members of the Club.

Example 3

When the Student Council conducted its newspaper drive, the Grade 7 class collected $19\frac{1}{2}$ kilograms, Grade 8 collected $26\frac{1}{8}$ kilograms, Grade 9 collected $22\frac{2}{3}$ kilograms. How many kilograms of newspapers did the three grades collect?

Solution: In this problem, all you need to do is find the total number of kilograms the three grades have collected.

So, Number of kg $= 19\frac{1}{2} + 26\frac{1}{8} + 22\frac{2}{3}$

$= 19 + 26 + 22 + \left(\frac{1}{2} + \frac{1}{8} + \frac{2}{3}\right)$

$= 67 + \left(\frac{12}{24} + \frac{3}{24} + \frac{16}{24}\right)$

$= 67 + \frac{31}{24}$

$= 67 + 1\frac{7}{24}$

$= 68\frac{7}{24}$ kg **Answer**

Example 4

Vicky has K3000 in the bank. She withdrew three–fourths of this amount to pay the tuition fee for her children. How much money was left in the bank?

Solution: Amount left in the bank = K3000 – $\frac{3}{4}$ of K3000

$$= K3000 - \frac{3}{4} \times K3000$$

$$= K3000 – K2250$$

$$= K750$$ **Answer**

Example 5

Ben had a cord $15\frac{3}{4}$ metres long. He cut it into pieces $1\frac{1}{8}$ metres long. How many pieces did he have?

Solution: The total length of the cord must be divided into $1\frac{1}{8}$ metres to get the number of pieces Ben had.

$$= 15\frac{3}{4} \div 1\frac{1}{8}$$

$$= \frac{63}{4} \div \frac{9}{8}$$ change to improper fractions

$$= \frac{63}{4} \times \frac{8}{9}$$ multiply by the reciprocal of $\frac{9}{8}$

$$= \frac{\overset{7}{\cancel{63}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{9}}}$$ cancel

$$= \frac{7 \times 2}{1 \times 1}$$ Simplify

$$= 14 \text{ metres}$$ **Answer**

NOW DO PRACTICE EXERCISE 6

Practice Exercise 6

Solve each of the following . Be sure the answer is reduced to its lowest term.

1. Baru has 5 bags of potato. Each bag holds $3\frac{2}{5}$ kilograms.
   a) What is the total weight of potatoes?

   b) How much do $3\frac{1}{2}$ bags weigh?

2. During her sister‟s birthday party Alu bought 25 m rolls of paper tablecloth from a party shop. If the tables are $2\frac{1}{4}$ metres long, how many tablecloths can be cut from the roll?

3. Kasa hikes $3\frac{1}{2}$ kilometers per day. At this rate, how many kilometers will he travel in $4\frac{3}{4}$ days?

4. The Hukahu family has two water tanks with $40\frac{5}{8}$ kilolitres of water in each of them.

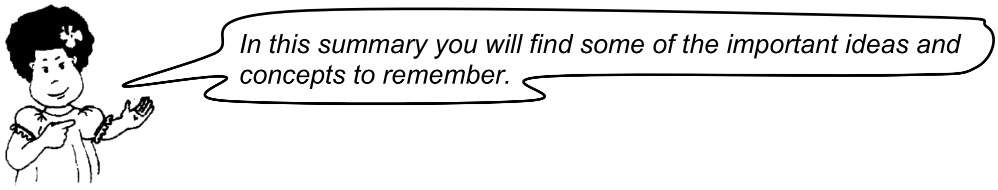
a) How many litres of water do they have?

b) If they use $2\frac{2}{3}$ kilolitres of water per week, how long will it take them to empty the tanks?

c) If there are four people in the Hukahu household, how much does each person use per week, on average?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

SUB-STRAND 1: SUMMARY



- To compare fractions with the same denominator, look at their numerators. The larger fraction is the one with the larger numerator.
- To compare fractions with different denominators, take the cross product. The first cross-product is the product of the first numerator and the second denominator. The second cross-product is the product of the second numerator and the first denominator. Compare the cross products using the following rules:
  a) If the cross-products are equal, the fractions are equivalent.
  b) If the first cross product is larger, the first fraction is larger.
  c) If the second cross product is larger, the second fraction is larger.
- If the fractions have the same denominator, their sum or difference is the sum or difference of the numerators over the denominator. We do not add or subtract the denominators. Reduce if necessary.
- If the fractions have different denominators:
  a) First, find the least common denominator.
  b) Then write equivalent fractions using this denominator.
  c) Add or subtract the fractions. Reduce if necessary.
- To multiply a fraction by a whole number, write the whole number as an improper fraction with a denominator of 1, and then multiply as fractions.
- When two fractions are multiplied, the result is a fraction with a numerator that is the product of the fractions' numerators and a denominator that is the product of the fractions' denominators.
- To multiply mixed numbers, convert them to improper fractions and multiply
- To divide a number by a fraction, multiply the number by the reciprocal of the fraction.
- The **reciprocal** of a fraction is obtained by switching its numerator and denominator. Note that when you multiply a fraction and it‟s reciprocal, the product is always 1.
- To divide mixed numbers, you should always convert to improper fractions, then multiply the first number by the reciprocal of the second

REVISE LESSONS 1 – 6. THEN DO SUB-STRAND TEST 1 IN ASSIGNMENT 1.

ANSWERS TO PRACTICE EXERCISES 1- 6

Practice Exercise 1

1. a) $\frac{5}{8}$ b) $\frac{12}{16}$ or $\frac{3}{4}$ c) $\frac{1}{3}$

2.

a)	$>$	f)	$>$
b)	$<$	g)	$>$
c)	$<$	h)	$<$
d)	$>$	i)	$>$
e)	$=$	j)	$<$

3. a) $\frac{2}{3}$ b) $\frac{1}{9}$ c) $\frac{5}{9}$ d) $\frac{3}{4}$

Practice Exercise 2

1. a) $\frac{3}{15}, \frac{4}{20}$.

   b) $\frac{6}{27}, \frac{8}{36}$

   c) $\frac{21}{28}, \frac{27}{36}$

   d) $\frac{24}{66}, \frac{28}{77}$

   e) $\frac{6}{18}, \frac{7}{21}$

2.

a)	$\frac{1}{5}$	f)	$\frac{7}{8}$
b)	$\frac{4}{7}$	g)	$\frac{9}{25}$
c)	$\frac{3}{20}$	h)	$\frac{1}{2}$
d)	$\frac{11}{20}$	i)	$\frac{1}{2}$
e)	$\frac{2}{5}$	j)	$\frac{7}{12}$

3. a) $2\frac{2}{3}$ f) $2\frac{1}{4}$

b) $3\frac{1}{2}$ g) $3\frac{3}{8}$

c) $1\frac{4}{5}$ h) $2\frac{2}{7}$

d) $1\frac{2}{7}$ i) $4\frac{3}{7}$

e) 2 j) $4\frac{1}{5}$

4. a) $\frac{102}{11}$ f) $\frac{21}{2}$

b) $\frac{23}{4}$ g) $\frac{31}{8}$

c) $\frac{25}{7}$ h) $\frac{36}{7}$

d) $\frac{13}{3}$ i) $\frac{35}{8}$

e) $\frac{4}{3}$ j) $\frac{12}{5}$

Practice Exercise 3

1. a) $1\frac{3}{20}$ e) $14\frac{2}{9}$

b) $\frac{5}{42}$ f) $24\frac{5}{9}$

c) $14\frac{1}{2}$ g) $7.\frac{2}{15}$

d) $4\frac{7}{9}$ h) 1

2. a) 2 pies b) $\frac{1}{10}$

3. $2\frac{11}{14}$

4. a) $\frac{27}{40}$ b) Gym = K20, book = K32, bank = K108

Practice Exercise 4

1. a) $\frac{4}{5}$ f) 10

   b) $1\frac{3}{7}$ g) 12

   c) $8\frac{1}{3}$ h) $5\frac{1}{3}$

   d) $\frac{10}{21}$ i) $\frac{1}{15}$

   e) $\frac{7}{64}$ j) $21\frac{3}{5}$

2. a) $19\frac{1}{4}$

   b) K166.40

   c) 17 kg

Practice Exercise 5

1. a) $\frac{3}{2}$ f) $\frac{3}{11}$

   b) 5 g) $\frac{1}{2}$

   c) $\frac{7}{2}$ h) $\frac{4}{21}$

   d) $\frac{8}{7}$ i) $\frac{5}{9}$

   e) 3 j) $\frac{9}{5}$

2. a) $4\frac{1}{2}$ e) $\frac{2}{7}$

   b) 15 f) $\frac{2}{11}$

   c) $1\frac{13}{35}$ g) $1\frac{7}{8}$

   d) 1 h) $\frac{5}{14}$

3. a) 41 blocks

   b) $8\frac{23}{24}$ bags

Practice Exercise 6

1. a) 17 kg

   b) $11\frac{9}{10}$ kg

2. 11 tablecloths

3. $16\frac{5}{8}$ km

4. a) $81\frac{1}{4}$ kilolitres

   b) $30\frac{15}{32}$ weeks

   c) $\frac{2}{3}$ kilolitres

END OF SUB-STRAND 1

SUB-STRAND 2

DECIMALS

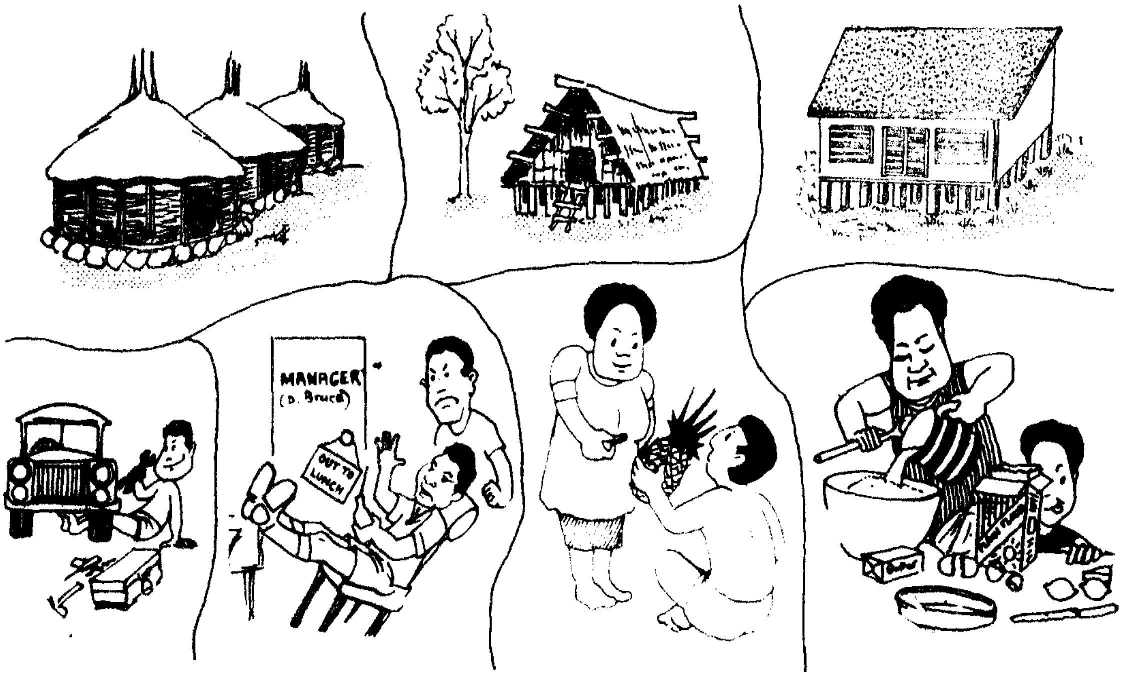
SUB-STRAND 2: DECIMALS

Introduction

In Grade 7 Mathematics, you learnt more about the decimal system and all the operations involving decimals. You also learnt to work out simple money and measurement problems using the basic operations.

This sub-strand is extending the work you did in Grade 7 Mathematics on addition, subtraction, multiplication and division of decimals to problem-solving in real life situations.

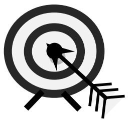
People live in many different places and work in many different jobs. At home and at work people confront different problems in real life situations. For example in business which requires a thorough knowledge of and skills on these basic operations.



Decimals are used in almost every walk of life. If you like to work in a trade store, in the bank and so on, then you must practice your basic operations.

Lesson 7: Decimals and Decimal Notation

You learnt about fractions and its application in real life situations in Sub-strand 1.



In this lesson, you will

- clearly define the meaning of decimals
- express given decimal numbers in exponential notation
- identify the place value of the digits of a decimal number

Do you remember the place values for whole numbers? For a four digit number, for example, the digits have the following values.

2 4 6 3 — ones
tens
hundreds
thousands

How do you read the four digit number above?

In writing a number, the value of each place is one-tenth $\left(\frac{1}{10}\right)$ of the value of the next place to the <u>left</u>.

Examples

$$\text{ones} = \frac{1}{10} \text{ of } 10$$

$$\text{tens} = \frac{1}{10} \text{ of } 100$$

$$\text{hundreds} = \frac{1}{10} \text{ of } 1000$$

$$\text{thousands} = \frac{1}{10} \text{ of } 10\,000$$

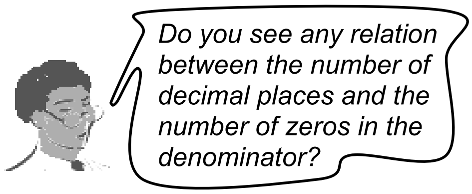
The values of the places to the <u>right</u> of the ones are determined by the same principle.

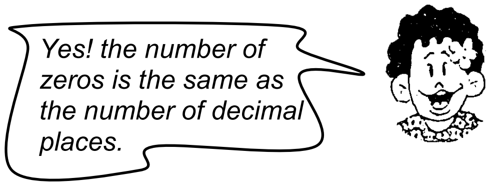
As we have seen, there are fractions like $\frac{1}{10}, \frac{13}{1000}, \ldots$ whose denominators are powers of ten. For these fractions, you already know that there is another way of writing them which makes use of our decimal place value system.

Let us review decimals in this lesson.

Decimals are another way of writing fractions and mixed numbers.

We write: $\frac{1}{10} = 0.1 \quad \frac{1}{100} = 0.01 \quad \frac{1}{1000} = 0.001$





This makes it easy to change these numerals from fraction to decimal and vice versa.

The following example shows the place value chart to the right of the ones place. The dot or decimal point is placed beside the ones place.

6 243. 645

$\frac{1}{10}$ x 1 or tenths or 10^{-1}

$\frac{1}{10}$ x $\frac{1}{10}$ or hundredths or 10^{-2}

$\frac{1}{10}$ x $\frac{1}{100}$ or thousandths or 10^{-3}

In Expanded Form,

$$6\,243.645 = 6\,243 + (6 \times \frac{1}{10}) + (4 \times \frac{1}{100}) + (5 \times \frac{1}{1000})$$

$$\text{or} = 6\,243 + \frac{6}{10} + \frac{4}{100} + \frac{5}{1000}$$

$$\text{or} = 6\,243 + (6 \times 10^{-1}) + (4 \times 10^{-2}) + (5 \times 10^{-3})$$

$$\text{or} = 6\,243 + 0.6 + 0.04 + 0.005$$

The complete decimal system we use can be summarized by the Place value chart. The decimal point is where there is a double line.

Thousands	hundreds	tens	ones	tenths	hundredths	thousandths	Ten thousandths
1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
6	2	4	3	6	4	5	

How does the decimal number above read?

In words, 6 243.645 is read as "**six thousand two hundred forty three and six hundred forty five thousandths".**

You will be reading very small decimals in the **millionths** and even **billionths** when you study about atoms in science, particularly in chemistry and physics.

As with the whole numbers, the value of a digit is 10 times more than the digit immediately to its right, and one-tenth as much as the value of a digit immediately to its left.

Thus, in 0.444, the underlined 4 is ten times more than the middle 4 and 100 times more than the shaded 4.

In a mixed decimal, the decimal point is read as "**and**".

Zeros added to the right of the decimal point, following the last digit do not change the value of the decimal. They are usually not used.

0.6 = 0.60 = 0.600 these all sound different when read, but all have the same value.

In reading or writing a number in decimal notation:

- **read or write the whole number**
- **the decimal point translates to "and"**
- **read or write the decimal part as though it were a whole number followed by the place value of the digit farthest to the right.**

Decimal fractions can be easily compared if they are expressed with the same denominators.

To compare the decimal fractions 0.6, 0.62 and 0.625, we rename them in terms of thousandths.

$$0.6 = 0.600 = \frac{600}{1000}$$

$$0.62 = 0.620 = \frac{620}{1000}$$

$$0.625 = 0.625 = \frac{625}{1000}$$

Therefore, for the decimal fractions 0.6, 0.62 and 0.625, the greatest is 0.625.

NOW DO PRACTICE EXERCISE 7

Practice Exercise 7

1. Use the Place Value Chart below to write the following numbers into their correct place value column.

a) 2.008
b) 3.12
c) 78.0002
d) 156.7071
e) 0.0006
f) 0.213
g) 5.67
h) 2.103
i) 123.5
j) 440.6

hundreds	tens	One s	.	tenths	hundredths	thousandths	Ten thousandths
			.				
			.				
			.				
			.				
			.				
			.				
			.				
			.				
			.				
			.				

2. What is the place value of **5** in the following numbers?

a) 548.3 ___________
b) 94.51 ___________
c) 5.482 ___________
d) 0.022359 ___________
e) 58.8 ___________
f) 65.9 ___________
g) 1.345117 ___________
h) 27.35 ___________
i) 0.000058 ___________
j) o.5321 ___________

3. Write the following numbers in expanded form.

a) 5.85 =
b) 21.8 =
c) 3.038 =
d) 8.346 =
e) 1.007 =

4. Express the following in correct decimal form.

a) $13 + \frac{5}{10}$

b) $8 + \frac{3}{10} + \frac{1}{100}$

c) $14 + \frac{8}{10} + \frac{4}{1000}$

d) $48 + \frac{7}{100} + \frac{4}{10\,000}$

e) $\frac{3}{10} + \frac{1}{100} + \frac{4}{1000} + \frac{4}{10\,000}$

---

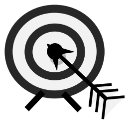
5. For each pair of decimals, write down the number which is bigger.

a) 0.543 or 5.43
b) 7.6 0r 7.06
c) 4.96 0r 4.69
d) 0.014 or 0.14
e) 7 hundredths or 0.007

CORRECT YOUR WORK. ANSWER ARE AT THE END OF SUB-STRAND 2.

Lesson 8: Problems involving Addition of Decimals

You have defined, read and written decimals in Lesson 7. You also learnt to find the value of a digit in a given decimal and which is bigger.



In this lesson, you will:

- add decimals with speed and accuracy
- solve problems in real life situations involving addition of decimals

You dealt with addition of decimals in Grade 7 Mathematics. In this discussion, you will revise the skills and extend your knowledge about addition of decimals.

When adding decimals, ensure that the digits are placed in their correct place value columns and that the decimal points are in line. It may be helpful to fill any empty decimal places with zeros. Hence, units of equal value will fall under the same column such as tenths, hundredths, thousandths and so on.

Example 1 Add 3.085, 12.314 and 94.65

Solution:

$$\begin{array}{r@{.}l}
3 & 085 \\
12 & 314 \\
94 & 650 \longrightarrow \text{0 is filled in after 5} \\
\hline
110 & 049
\end{array}$$

same column

Example 2 Add 14 .37 + 1.66 + 23 . 8

Solution:

$$\begin{array}{r@{.}l}
14 & 37 \\
1 & 66 \\
23 & 80 \longrightarrow \text{0 is filled in after 8} \\
\hline
39 & 83
\end{array}$$

same column

Example 3 Add 3.13 + 5.4 + 6.26

Solution:

$$\begin{array}{r@{.}l}
3 & 13 \\
5 & 40 \longrightarrow \text{0 is filled in after 4} \\
6 & 26 \\
\hline
14 & 79
\end{array}$$

same column

Now recap what you have discussed in the examples.

To add decimals

- **write the numbers so that the decimal points line up vertically.**
- **add as whole numbers**
- **place the decimal point of the answer so that it lines up vertically with the decimal points in the addends.**
- **zeros may be added to the right end of any number so that it will be easier to keep the number aligned in columns.**

Now study this problem.

Mrs. Tiu needs 2.6 metres of cloth for the dress of one of her daughters and 3.2 metres of cloth for the dress of another daughter. How many metres of cloth does she need for her daughter‟s dresses?

Solution: The problem requires addition in order to find the total number of metres of cloth Mrs Tiu needs for her daughters‟ dresses.

Add 2.6 and 3.2

$$\begin{array}{r} 2.6 \\ +\ 3.2 \\ \hline 5.8 \end{array}$$

Mrs Tiu needs **5.8 metres** of cloth

There are many problems in real life situations such as this where addition is involved.

Here are other examples.

Example 1

Find the total weight for 45.75 kilograms, 16.25 kilograms and 100.2 kilograms.

Solution: Add 45.75 + 16.25 + 100.2

$$\begin{array}{rl} 45.75 & \\ 16.25 & \\ \underline{100.20} & \longrightarrow \text{0 filled in after 2} \\ 162.20 & \end{array}$$

Therefore, total weight is **162.20** kilograms

Example 2

On the first day of their holiday, the Kibai family drove 187.4 km. They drove 147.3 km on the second day and 208.25 km on the third day. How many kilometres did they drive during the entire three days?

Solution: Add 187.4 km + 147.3 km + 208.25 km

$$\begin{array}{r} 187.40 \\ +\ 147.30 \\ \underline{208.25} \\ 542.95 \end{array}$$

0s filled in after 4 and 3

Therefore, the Kibai family drove **542.95 km** in three days

Example 3

Yara Fe went to the supermarket and purchased the following items:

a packet of jelly beans for K1.86
a block of chocolate for K3.28
a bottle of soft drink for K1.87
a packet of chips for K2.79

How much did Yara Fe spend?

Solution: Add the cost of each item.

$$\begin{array}{r} 1.86 \\ 3.28 \\ +\quad 1.87 \\ \underline{2.79} \\ 9.80 \end{array}$$

Therefore, Yara Fe spent **K9.80.**

NOW DO PRACTICE EXERCISE 2

Practice Exercise 8

---

1. Add these decimal numbers.

a) $\begin{array}{r} 12.909 \\ +\ \underline{93.007} \end{array}$

b) $\begin{array}{r} 127.3 \\ \underline{+45.6} \end{array}$

c) $\begin{array}{r} 237.72 \\ \underline{+\ 56.91} \end{array}$

d) $\begin{array}{r} 23.21 \\ \underline{+13.54} \end{array}$

e) $\begin{array}{r} 34.95 \\ \underline{+6.86} \end{array}$

---

2. Add the following decimals correctly.

a) 0.65 + 3.98 + 5.12
b) 12.763 + 45.601
c) 104.69 + 23.28
d) 129.87 + 42.9
e) 14.89 + 342.09 + 5.01 + 12.77
f) 160.876 + 2.801 + 0.083
g) 1.05 + 4.99 + 120.12
h) 10.6 + 250.9 + 19.3
i) 3018.34 + 136.8 + 568.01
j) 1000.74 + 44.82 + 0.07 + 1.7

Refer to the information below to answer Questions 3 and 4.

Aaron took 5 parcels to the Waigani Post Office. The parcels weighed 2.75 kg, 0.58 kg, 0.27 kg, 1.8 kg and 0.95 kg.

3. What was the total weight of his parcels?

4. What is the combined weight of the heaviest and the lightest parcel?

5. Nancy needs to buy gear for her hiking adventure. She will need to purchase a good pair of boots, a watch, a compass, a vest, a pair of khaki pants and a cotton shirt. She already has all the other necessary hiking gear.

Listed below are the costs of the different items she wanted to buy.

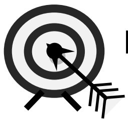
Pants	K44.95
boots	K125.99
shirt	K64.37
vest	K51.21
compass	K99.89
watch	K1150.27

Find the total cost of the items.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2

Lesson 9: Problems Involving Subtraction of Decimals

In the previous lesson, you learnt to work out problems involving addition of decimals.



In this lesson, you will:

- subtract decimals with speed and accuracy
- solve problems in real life situations involving subtraction of decimals

If you know how to add decimals, then you can subtract decimals. The rule used in addition applies to subtraction.

When subtracting decimals, check that the digits are placed in their correct place value columns and that the decimal points are in line just as you do in addition.

You will also need to fill empty decimal places with zeros. The decimal point should appear in the answer in the correct place value column, as in the question.

Here are subtraction examples.

Example 1

Subtract 27. 583 – 0.2

To subtract these numbers, first arrange the terms vertically, aligning the decimal points in each term. You can add zeroes to the right of the decimal point, to make it easier to align the columns. Then subtract the columns working from the right to the left, putting the decimal point in the answer directly underneath the decimal points in the terms. Check your answer by adding it to the second term and making sure it equals the first.

Solution:

$$\begin{array}{r} 27.583 \\ -\ \underline{0.200} \\ 27.383 \end{array} \longleftarrow \text{0s filled in after 2}$$

same column

Check:

$$\begin{array}{r} 27.383 \\ +\ \underline{0.200} \\ 27.583 \end{array}$$

Example 2

Subtract 125.2684 from 436.1052

Solution:

$$\begin{array}{r} 436.1052 \\ -\ \underline{125.2684} \\ 310.8368 \end{array}$$

Check:

$$\begin{array}{r} 310.8368 \\ +\ \underline{125.2684} \\ 436.1052 \end{array}$$

Now you recap what you have discussed in the examples.

To subtract decimals

- **Put the numbers in a vertical column, aligning the decimal points.**
- **Subtract each column, starting on the right and working left. If the digit being subtracted in a column is larger than the digit above it, "borrow" a digit from the next column to the left.**
- **Place the decimal point in the answer directly below the decimal points in the terms.**
- **Check your answer by adding the result to the number subtracted. The sum should equal the first number.**

Now look at the application of subtraction of decimals in problems involving decimals.

Example 1

Ten years ago, shares in a particular bank were priced at K9.35. They are now priced at K31.12.

By how much has the price of these shares risen?

Solution: Subtract K31.12 and K9.35 to get rise in price of shares.

$$\begin{array}{r} 31.12 \\ -\ \underline{9.35} \\ 21.77 \end{array} \qquad \text{Check:} \quad \begin{array}{r} 21.77 \\ +\ \underline{9.35} \\ 31.12 \end{array}$$

Therefore, the price of shares has risen by **K21.77**.

Example 2

On his trip, Kopi completed a trip of 1382 km. When he arrived home the odometer showed 40 153.2 km. What was the reading before the trip began?

Solution: Subtract 1382 km from 40 153.2 km to get the reading before the trip

$$\begin{array}{r} 40153.2 \\ \underline{-\quad 1382.0} \\ 38771.2 \end{array} \qquad \text{check:} \qquad \begin{array}{r} 38771.2 \\ \underline{+\quad 1382.0} \\ 40153.2 \end{array}$$

Therefore, the reading before the trip began is **38771.2 km**

Example 3

Mathias earns K10 372.57 a month as a logistics manager. His monthly deductions amount to K3201.07. How much money will Mathias receive after deductions?

Solution: Subtract K3201.07 from K10 372.57

$$\begin{array}{r} 10\ 372.57 \\ \underline{-\ 3\ 201.07} \\ 7171.50 \end{array} \qquad \text{Check:} \qquad \begin{array}{r} 7171.50 \\ +\ \underline{3201.07} \\ 10372.57 \end{array}$$

Therefore: Mathias will receive **K7171.50** after deductions.

Example 4

Stopping at a petrol station, Gibo gave the attendant K300 for oil which cost K37.85 and petrol worth K255.65. How much was his change if any?

Solution: First add K37.85 and K255.65, then, subtract the sum from K300 to get the change if there is any.

so, $$\begin{array}{r} 37.85 \\ \underline{+\ \ 255.65} \\ 293.50 \end{array}$$

Subtract the 293.50 from K300

$$\begin{array}{r} 300.00 \\ \underline{-\ \ 293.50} \\ 6.50 \end{array}$$

Therefore, Gibo received **K6.50** change.

NOW DO PRACTICE EXERCISE 9

Practice Exercise 9

1. Subtract the following decimal numbers.

   a) 0.95 – 0.24
   b) 784.98 – 56.89
   c) 26.26 – 12.121
   d) 145.12 – 29.7
   e) 302.9 – 56.89
   f) 4.75 – 1.681
   g) 6.005 – 0.987
   h) 37.32 – 22.9006
   i) 521. 642 – 232.905
   j) 0.975 – 0.866

2. Subtract these decimals.

   a) $\begin{array}{r} 167.32 \\ -\ \ 94.96 \\ \hline \end{array}$ b) $\begin{array}{r} 140.39 \\ -\ \ 23.20 \\ \hline \end{array}$ c) $\begin{array}{r} 1081.34 \\ -\ 568.20 \\ \hline \end{array}$

   d) $\begin{array}{r} 1164.9 \\ -\ 246.86 \\ \hline \end{array}$ e) $\begin{array}{r} 219.87 \\ -\ \ 32.44 \\ \hline \end{array}$

3. What is the difference between K143.35 and K 116.25?

4. What is the difference between 121.05 metres and 3.76 metres?

5. How much bigger is 67.05 than 37.07?

6. Before starting their holiday, the Gairo family recorded their car"s odometer reading. They noted it again at the end of each day of travelling. The odometer readings, in kilometres, are given in the table below:

	Odometer reading
Start	108 372.7
Monday	108 818.8
Tuesday	109 391.3
Wednesday	109 900.5

a) Calculate the distance travelled on Monday.

b) Calculate the distance travelled on Tuesday.

c) Calculate the distance travelled on Wednesday.

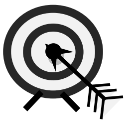
d) On which day did they travel the farthest?

e) What was the total distance travelled?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2

Lesson 10: Problems Involving Multiplication of Decimals

In the previous lessons, you learnt to work out problems involving addition and subtractions of decimals.



In this lesson, you will:

- multiply decimals with speed and accuracy
- solve problems in real life situations involving multiplication of decimals

Since decimals are fractions, you can consider multiplication of decimals based on the concept of multiplication of fractions. Thus, to multiply decimals you change each decimal factor to fractional form first. Then multiply the resulting fractions and convert the product back to decimal form. Recall that the product of two fractions is a fraction whose numerator and denominator are respectively, the product of the numerators and the product of the denominators of the fractions multiplied.

The following examples will show you the process of finding the product of decimals clearly.

1. $0.3 \times 0.4 = \frac{3}{10} \times \frac{4}{10}$

   $= \frac{12}{100}$

   $= 0.12$

   Therefore, **0.3 x 0.4 = 0.12**

2. $0.31 \times 0.5 = \frac{31}{100} \times \frac{5}{10}$

   $= \frac{155}{1000}$

   $= 0.155$

   Therefore, **0.31 x 0.5 = 0.155**

3. $0.16 \times 0.23 = \frac{16}{100} \times \frac{23}{100}$

   $= \frac{368}{10000}$

   $= 0.0368$

   Therefore, **0.16 x 0.23 = 0.0368**

4. $10.9 \times 0.64 = 10\frac{9}{10} \times \frac{64}{100}$

$= \frac{109}{10} \times \frac{64}{100}$

$= \frac{6976}{1000}$

$= 6.976$

Therefore, **10.9 x 0.64 = 6.976**

5. $9.3 \times 6.2 = 9\frac{3}{10} \times 6\frac{2}{10}$

$= \frac{93}{10} \times \frac{62}{10}$

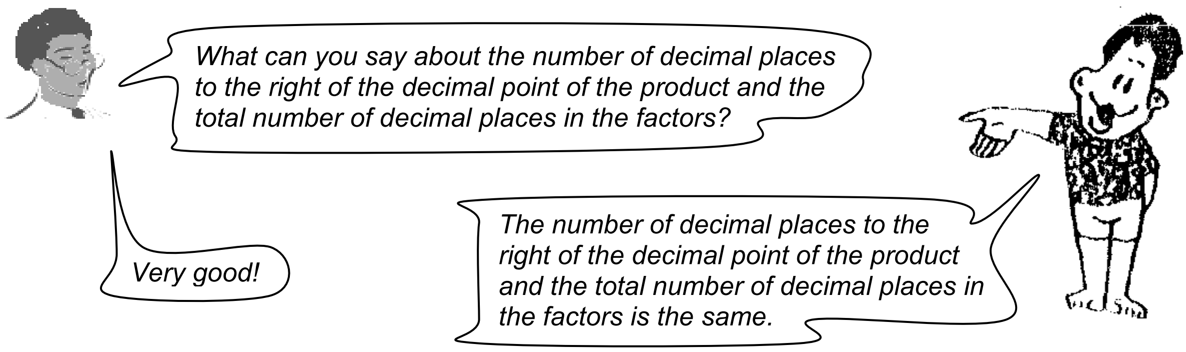
$= \frac{5766}{100}$

$= 57.66$

Therefore, **9.3 x 6.2 = 57.66**

Using the above results, we have formed the table below.

	First factor	Second factor	Product	Number of Decimal Places		
				In the First Factor	In the Second Factor	In the Product
1	0.3	0.4	0.12	1	1	2
2	0.31	0.5	0.155	2	1	3
3	0.16	0.23	0.0368	2	2	4
4	10.9	0.64	6.976	1	2	3
5	9.3	6.2	57.66	1	1	2



We can now outline the following rule for multiplying decimals.

See next page.

To multiply decimals

- **set out the question as a multiplication problem.**
- **multiply the decimal numbers as if they are whole numbers**
- **count the number of decimal places in the first factor and in the second factor**
- **Place the decimal point in the product so that the number of places to the right of the decimal point in the factors and in the product is the same.**
- **Zero may be added to the left of the product to allow for the correct number of decimal places.**

Here are some more examples.

Example 1

Multiply 6. 9 x 3

```
Solution:    6.9    (1 decimal place)
           x   3
            20.7
```

There is 1 decimal place in the factors, so the answer needs 1 decimal place.

Example 2

Find the product of 25.49 x 0. 5

```
Solution:    25.48    (2 decimal places)
             x  0.5   (1 decimal place)
            12.745
```

There are 3 decimal places in the factors, so the answer must have 3 decimal places.

Example 3

What is 50.8076 times 23. 569?

```
Solution:      50.8076    (4 decimal places)
            x  23. 569    (3 decimal places)
               4572684
              3048456
             2540380
            1524228
           1016152
          1197.4843244
```

There are 7 decimal places in the factors, so the answer needs 7 decimal places.

Now, let us work out problems in real life situations involving multiplication of decimals using the skills learned.

Example 1

A 2 L ice cream container sells for K27.95 each. How much will 8 containers cost?

Solution: Multiply the price of each ice cream container by 8

so,
$$\begin{array}{r} 27.95 \\ \underline{\times\ 8} \\ 223.60 \end{array}$$

Therefore, 8 containers of ice cream cost **K223. 60**

Example 2

A can of beer contains 9.2 grams of carbohydrates. How many grams of carbohydrates are there in 6 cartons of beer if each carton has 24 cans in it?

Solution: Multiply 24 cans by 6 cartons to get the total number of cans

$24 \times 6 = 96$

Then multiply the number of cans by the number of carbohydrates per can.

$$\begin{array}{r} 9.2 \\ \underline{\times\ 96} \\ 883.2 \end{array}$$

Therefore, the amount of carbohydrates in a carton of beer is **883.2 grams**

Example 3

Ike is fertilizing his vegetable garden. He uses 5.6 grams of fertilizer per square yard. The garden measures 72.5 square metres. How many grams of fertilizer does Ike need for his garden?

Solution: To find the total amount of fertilizer needed, multiply the measure of the garden by the amount of fertilizer needed per square metre.

$60.5 \times 5.6 = 338.8$ grams

NOW DO PRACTICE EXERCISE 10

Practice Exercise 10

1. Give the product of the following.

a) 56.6 x 4.82 b) 9.12 x 7.53 c) 6.8 x 8.6

d) 0.604 x 0.9 e) 1.5 x 0.02

2. Identify the number of decimal places there are in the answer to each of the following by calculating.

a) 6.37 x 2.72	b) 8.079 x 0.85	c) 47.857 x 3.55	d) 675.82 x 7.85
e) 9.578 x 0.0079	f) 45.078 x 6.019	g) 9.0075 x 3.005	h) 8.46 x 0.507

3. Problem Solving

a) If bones form 0.18 of a person‟s weight, what is the weight of the bones of a person who is 46.5 kg?

Answer:

b) Ball pens cost K3.15. How many can you buy with K20?

Answer:

c) A packet of Jasmine rice cost K4.95. How much will 20 packets cost?

Answer:

---

d) How much income does a lottery agent collect from the sale of 100 000 lottery tickets at K22.50 each?

Answer:

---

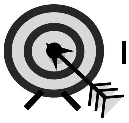
e) Troy earns K6.32 per hour for each of the first 36 hours he works in a week. He earns K9.50 per hour for each additional hour he works overtime in the same week. How much money will he earn if he works 52 hours in one week?

Answer:

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2.

Lesson 11: Problems Involving Division of Decimals

In the previous lessons, you learnt to work out problems involving multiplication of decimals.



In this lesson, you will:

- divide decimals with speed and accuracy
- solve problems in real life situations involving division of decimals

Dividing is the most challenging of the four basic operations. In fact, you have to use subtraction and multiplication in order to divide, and you also have to be pretty good at rounding and estimating. Many students have trouble with division.

Dividing decimals is almost the same as dividing whole numbers, except you use the position of the decimal point in the dividend to determine the decimal places in the result.

In Grade 7, you learnt how to divide decimals. Now look at the rules again.

To divide decimal numbers:

1. **If the divisor is not a whole number:**
   - **Move the decimal point in the divisor all the way to the right (to make it a whole number).**
   - **Move the decimal point in the dividend the same number of places.**
2. **Divide as usual. If the divisor doesn't go into the dividend evenly, add zeroes to the right of the last digit in the dividend and keep dividing until it comes out evenly or a repeating pattern shows up.**
3. **Position the decimal point in the result directly above the decimal point in the dividend.**
4. **Check your answer: Use the calculator and multiply the quotient by the divisor. Does it equal the dividend?**

Let us work through an example involving division of decimals.

Example 1 Find this quotient: 16.9 ÷ 6.5

Solution:

Step 1: Show the division like this:-

$$6.5\,\overline{)\,16.9}$$

Step 2: Move the decimal point one place to the right, which makes the divisor a whole number. Also move the decimal point in the dividend one place to the right.

$$65.\,\overline{)\,169.}$$

Step 3: Divide as whole numbers. 65 goes into 169 two times with 39 left over.

$$\begin{array}{r} 2 \\ 65\,\overline{)\,169.} \\ -\underline{130} \\ 39 \end{array}$$

Step 4: To continue dividing, add a zero to the right of the decimal point in the dividend. Then bring down the zero, and add it to the end of 39, making it 390.

$$\begin{array}{r} 2 \\ 65\,\overline{)\,169.0} \\ -\underline{130} \\ 390 \end{array}$$

Step 5: 65 goes into 390 six times. We write a 6 above the zero in the quotient and put the decimal point just above the decimal point in the dividend.

$$\begin{array}{r} 2.6 \\ 65\,\overline{)\,169.0} \\ -\underline{130} \\ 390 \\ -\underline{390} \\ 0 \end{array}$$

To check our answer, we multiply the quotient by the divisor and make sure it equals the dividend.

$$\begin{array}{r} 2.6 \\ \underline{\times\ \ 6.5} \\ 130 \\ \underline{+\ 156} \\ 16.90 \end{array}$$

Example 2

Find the quotient: 55.318 ÷ 33.4

Solution: 55.318 ÷ 33.4 ⟶ $33.4\overline{)54.3418}$ Write in standard form.

$334.\overline{)543.418}$ Move decimal point in divisor and dividend.

$$\begin{array}{r|l} & \ \ 1.627 \\ \hline 334. & 543.418 \\ & -334 \\ \hline & \ \ 2094 \\ & -2004 \\ \hline & \quad\ \ 901 \\ & \quad -668 \\ \hline & \quad\ \ 2338 \\ & \quad -2338 \\ \hline & \qquad\quad 0 \end{array}$$

Keep dividing until quotient repeats or comes out evenly.

Add zeros on right of dividend as needed.

The quotient is 1.627. Answer

Now, let us work out problems in real life situation involving division of decimals using the skills learned.

Example 1

Samuel has 95.7 kilograms of vegetables and 137.7 kilograms of fruits to be delivered to three customers in equal amounts. How many kilograms of vegetables and how many kilograms of fruits would each customer get?

Solution: To find the number of kilograms for each customer we divide each of the two amounts by three.

So, for vegetables $= \frac{95.7}{3} = 31.9$ kilograms

for fruits $= \frac{137.7}{3} = 45.9$ kilograms

Therefore, each customer gets **31.9 kg** of vegetables and **45.9 kg** of fruits.

Example 2

A container has 5.65 litres of cordial. If this is divided equally amongst 5 students, how much would each student get?

Solution: To find how much each student gets, divide the total amount of cordial in the container by the number of students.

So, $\frac{\text{amount of cordial}}{\text{no. of students}} = \frac{5.65}{5} = 1.13$ litres

Therefore, each student gets **1.13 litres** of cordial.

Example 3

A building is planned to be 8.5 metres wide to fit the land available. How long should the building be to have an area of 137.7 square metres?
Hint: (Length = area ÷ Width)

Solution: This problem asked for the length of the building given its area and its width.

To find the length, we divide the area by its width.

So, $\text{Length} = \dfrac{\text{area of building}}{\text{width of building}} = \dfrac{137.7}{8.5} = 16.2 \text{ metres}$

Therefore, the building should be **16.2 metres** long to fit into the land.

Example 4

A lottery prize of K32 472.75 is to be divided equally among 3 winners. How much does each one receive?

Solution: As stated in the problem, K32 472. 75 is to be divided equally into 3.

So, $\dfrac{\text{K32 472.75}}{3} = \text{K10 824.15}$

Therefore each winner receives **K10 824.15**.

Example 5

A book is 6.6 mm thick. How many of the same books may be stored on a shelf 1.65 m long?

Solution: First, we convert 1.65 m to mm = 1.65 x 1000 mm = 1650 mm

Divide the result by 6.6 mm to find the number of books that can be stored on the shelf.

So, 1650 ÷ 6.6 = 250

Therefore, **250 books** can be stored on the shelf.

NOW DO PRACTICE EXERCISE 11

Practice Exercise 11

1. Find the quotient.

a) $25\overline{)3.5}$

e) $0.21\overline{)33.551}$

b) $5\overline{)28.35}$

f) $0.04\overline{)25.648}$

c) $0.25\overline{)325}$

g) $0.3\overline{)68.76}$

d) $0.05\overline{)2.835}$

h) $0.5\overline{)20.45}$

e) $2.5\overline{)3.25}$

i) $8\overline{)64.16}$

2. Problem Solving

a) A container full of pawpaws weighed 10.5 kilograms. if each pawpaw weighs 0.70 kilogram, how many pieces of pawpaw were in the container?

---

b) Another container weighed 20.75 kilograms. If it contained 100 pieces of apples, how much did each apple weigh?

Refer to the information below to answer Questions c and d.

Kopi works as a delivery boy for a fruit and vegetable dealer on weekends.

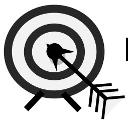
d) The distance from the orchard to the market is about 63.5 kilometres. If Kopi travels at the rate of 40.5 kilometres per hour, how many minutes would it take to get from the orchard to the market?

e) On a particular weekend, he earned K103. What was his daily wage?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OFSUB-STRAND 2.

Lesson 12: Applications of Decimals in Real Life Situations

In the previous lessons, you learnt to work out problems involving the four operations on decimals.



In this lesson, you will:

- solve and work out a mixture of problems in real life situations involving decimals.

There is a mixture of problems in real life situations involving decimals. We often encounter decimals in our daily dealings whether we are at home, in school, in the office, at the market and so on. These include:

1. Money: For example: 97 kina and 36 toea which is usually written as K97.36
2. Measurement of Length: For example: 6 metres and 55 centimetres which is usually written as 6.55 m
3. Measurement of Capacity: For example: 32 litres and 253 millilitres which is usually written as 32.253 L

Your skills at adding, subtracting, multiplying and dividing decimals are needed to work out the problems in this lesson. You need to read the problems carefully, twice, three times or even ten times to know and understand **what is asked** and **what is given** in the problem and **what operation** can be used or applied to find the answer.

Let us study the following examples.

Example 1

Paul sawed an 8 metre wooden beam and removed 4.22 metres. What is the length of the remaining piece?

Solution: What is given: Total length of wooden beam = 8 metres
Length of wood removed = 4.22 metres

What is asked: Length of remaining piece = ________

The problem requires subtraction.

To find the length of the remaining piece, we subtract 4.22 metres from 8 metres.

Hence, we have

Length remains = Length of beam of wood - Length taken off

So, Length remains = 8 m – 4.22 m

= 3.78 m

Therefore the remaining piece is **3.78 metres**.

Example 2

Five boys earned K814.75 in one week for selling newspapers. Ivan earned K101.25; Ben, K96.00; Eugene, K82.50 and Randal, K50.75. How much did Frank earn?

Solution: The problem is asking for the amount that Frank, who is the fifth boy, earned in one week.

To work it out, we need to add all the amounts earned by Ivan, Ben, Eugene and Randal. Then, the result will be subtracted from the total amount earned by the five boys, which is K814.75.

Hence, we have

Amount earned by Frank = Total Amt. earned by 5 boys – Sum of Amt. earned by 4 boys

= K814.75 – (K101.25 + K96 + K82.50 + K50.75)

= K814.75 – K330.50

= K484.25

Therefore, Frank earned **K484.25** in a week.

Example 3

Rhoda went to the shop and bought the following food items:

$\frac{1}{2}$ kg of pork mince for K13.50
2 kg of pork belly bone for K56.75
1 kg of local tomato for K12.35
1 kg of pork shoulder chopped for K31.95
1 kg imported onions for K5.50

a) Find the total cost of her shopping?
b) Calculate Rhoda‟s change if she paid two K100 notes.

Solution:

a) To find the total cost of Rhoda‟s shopping, we use addition.

Total cost = K13.50 + k56.75 + K12.35 + K31.35 + K5.50

= K119.45

b) To find Rhoda‟s change from two K100 notes, we subtract the total cost of her shopping from the K200.

Change = K200 – K119.45
= K80.55

Therefore: Rhoda‟s change from two K100 notes is **K80.55**.

Example 4

Calculate the price of a pair of sports shoes at K56.45 if a 10% value added tax (VAT) was added?

Solution: a) First, we find the 10% Value Added Tax added.

The quickest way of getting 10% of a price is to shift or move the decimal point one place to the left.

Hence, we have 10% of K56.45 = K5.645

b) Add the Amount of VAT to the original price which is K56.45

K56.45 + K5.645 = K62.095 rounded to the nearest toea

= K62.10

Therefore, the price of the pair of shoes is **K62.10**.

NOW DO PRACTICE EXERCISE 12.

Practice Exercise 12

Read the Worded Problems carefully and solve by applying the correct operations.

1. Mrs. Loa bought K245.80 worth of groceries and K37.50 worth of vegetables.

   How much change did she receive from her K1000?

2. A school canteen sold hot cakes at K7.50 each.

   How many hot cakes were sold if the sale was K240?

3. Kakas bought 12.5 kilograms of sugar at K4.80 per kilogram, 10.5 kilograms of flour at K3.80 per kilogram and 8 cans of condensed milk at K4.60 per can.

   How much did he pay all together?

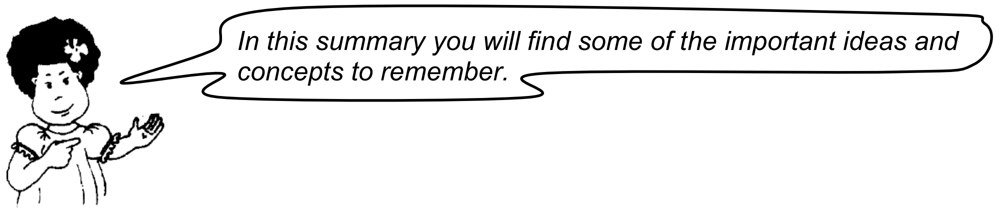
4. During the first day of his vacation, Jonah drove 147.3 km; 178.9 km on the second day and 202.75 km on the third day.

   How many kilometres did he drive during his entire vacation?

5. Stopping at a gasoline station, Ron gave the attendant K400 for oil which cost K47.85 and gasoline worth K345.65. How much was his change, if any?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2.

SUS-STRAND 2: SUMMARY



- Decimals are another way of writing fractions and mixed numbers.
- In reading or writing a number in decimal notation:
    1. read or write the whole number
    2. the decimal point translates to "and"
    3. read or write the decimal part as though it were a whole number followed by the place value of the digit farthest to the right.
- To add decimals, write the numbers so that the decimal points line up vertically and add as whole numbers. Place the decimal point of the answer so that it lines up vertically with the decimal points in the addends.
- To subtract decimals, put the numbers in a vertical column, aligning the decimal points. Subtract each column, starting on the right and working left. If the digit being subtracted in a column is larger than the digit above it, "borrow" a digit from the next column to the left. Place the decimal point in the answer directly below the decimal points in the terms.
- To multiply decimals, multiply the decimal numbers as if they are whole numbers. Count the number of decimal places in the first factor and in the second factor. Place the decimal point in the product so that the number of places to the right of the decimal point in the factors and in the product is the same. Zero may be added to the left of the product to allow for the correct number of decimal places.
- To divide decimal numbers:
    1. If the divisor is not a whole number:

       Move the decimal point in the divisor all the way to the right (to make it a whole number).

       Move the decimal point in the dividend the same number of places.
    2. Divide as usual. If the divisor doesn't go into the dividend evenly, add zeroes to the right of the last digit in the dividend and keep dividing until it comes out evenly or a repeating pattern shows up.
    3. Position the decimal point in the result directly above the decimal point in the dividend.
- To solve problems involving decimals, read the problem carefully, twice, three times or even ten times to know and understand what is being asked and what is given to know what operation can be used or applied to find the answer.

REVISE LESSONS 7 – 12. THEN DO SUB-STRAND TEST 2 IN ASSIGNMENT 1.

ANSWERS TO PRACTICE EXERCISES 7-12

Practice Exercise 7

1.

hundreds	tens	ones	.	tenths	hundredths	thousandths	Ten thousandths
		2	**.**	**0**	**0**	**8**	
		3	**.**	**1**	**2**		
	7	**8**	**.**	**0**	**0**	**0**	**2**
1	**5**	**6**	**.**	**7**	**0**	**7**	**1**
			.				**6**
			.	**2**	**1**	**3**	
		5	**.**	**6**	**7**		
		2	**.**	**1**	**0**	**3**	
1	**2**	**3**	**.**	**5**			
4	**4**	**0**	**.**	**6**			

2. a) hundreds f) ones
   b) tenths g) thousandths
   c) ones h) hundredths
   d) hundred thousandths i) hundred thousandths
   e) tens j) tenths

3. a) $5 + \frac{8}{10} + \frac{5}{100}$

   b) $21 + \frac{8}{10}$

   c) $3 + \frac{3}{100} + \frac{8}{1000}$

   d) $8 + \frac{3}{10} + \frac{4}{100} + \frac{6}{1000}$

   e) $1 + \frac{7}{1000}$.

4. a) 13.5 c) 14.804 e) 0.3144
   b) 8.31 d) 48.0704

5. a) 5.43
   b) 7.6
   c) 4.96
   d) 0.14
   e) 7 hundredths

Practice Exercise 8

1. a) 105.916
   b) 172.9
   c) 294.63
   d) 36.75
   e) 41.81

2. a) 9.75 f) 163.760
   b) 58.364 g) 126.16
   c) 127.97 h) 280.8
   d) 172.77 i) 3723.15
   e) 374.76 j) 1047.33

3. 6.35 kg
4. 3.02 kg
5. 1536.68

Practice Exercise 9

1. a) 0.71 f) 3.069
   b) 728.09 g) 5.018
   c) 14.139 h) 14.4194
   d) 115.42 i) 288.737
   e) 246.01 j) 0.109

2. a) 72.36
   b) 117.19
   c) 513.14
   d) 918.04
   e) 187.43

3. K27.10
4. 117.29 m

5. 29.98

6. a) 446.1 km
   b) 572.5 km
   c) 509.2 km
   d) Tuesday
   e) 1527.8 km

Practice Exercise 10

1. a) 272.812
   b) 68.6736
   c) 58.48
   d) 0.5436
   e) 0.030

2. a) 4 dec.p. e) 7 dec.p
   b) 5 dec.p. f) 6 dec.p
   c) 5 dec.p. g) 7 dec.p
   d) 4 dec.p h) 5 dec.p

3. a) 8.37 kg
   b) 6 pens
   c) K99
   d) K2 250 000
   e) K3790.52

Practice Exercise 11

1. a) 0.14 f) 159.76
   b) 5.67 g) 641.2
   c) 1300 h) 229.2
   d) 56.7 i) 40.9
   e) 1.3 j) 8.02

2. a) 15 pieces
   b) 0.2075 kg or 207.5 g
   c) 1.57 minutes
   d) K51.5

Practice Exercise 12

1. K716.70

2. 32 hotcakes

3. K136.70

4. 528.95 km

5. K6.50

END OF SUB-STRAND 2

SUB-STRAND 3

PERCENTAGES

SUB-STRAND 3: PERCENTAGES

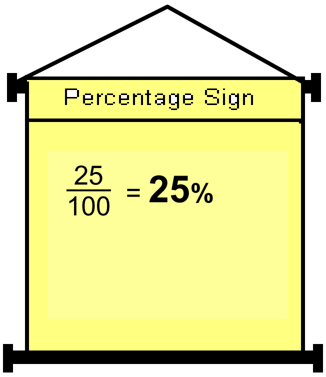
Introduction

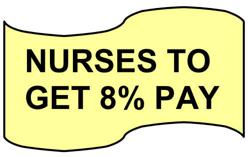
This is the third sub-strand of the Grade 8 Mathematics Strand 1. This Sub-strand reviews the meaning and usage of percentages.

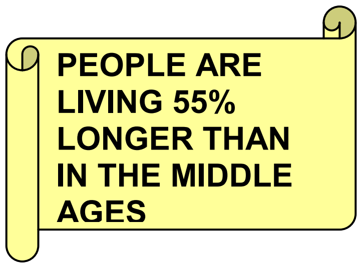
Percentages are special fractions, all of which have the **same denominator, 100**. The word **percentage** came from the Latin word, **per centum**, meaning **per hundred**.

For example, 25% is read as "twenty-five percent" and means $\frac{25}{100}$

Look at the charts.







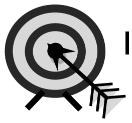
Can you see how the percentage sign (%) may have been invented to represent the denominator 100?

Percentages are used in all walks of life. Therefore we all need to understand percentages and be able to perform calculations involving percentage.

In this Sub-strand, you will:
- review the meaning and the usage of percentages.
- solve problems that involve percentages.

Lesson 13: Review on Percentages

You learnt about percentages in Grade 7 Mathematics.



In this lesson, you will:

- revise percentages
- change percentages to fractions and decimals or vice versa.

You learnt that fractional numbers may be expressed in many ways. For example, $\frac{2}{4}$, $\frac{50}{100}$ and 0.5 are some names for one-half.

You also learnt that percent is used to name fractional numbers.

Let us revise what percent or percentage means.

Percent means out of a hundred or per hundred. The symbol for percent is %.

Examples

1. 12 percent means 12 per hundred or 12 %
2. 103 percent means 103 per hundred or 103 %.
3. $6\frac{1}{4}$ percent means $6\frac{1}{4}$ per hundred or $6\frac{1}{4}$ %

The term percent implies a denominator of 100. As a fraction, a percent (%) means $\frac{1}{100}$, and as a decimal it means 0.01.

100% is equivalent to 1 or $\frac{100}{100}$. If a percent is less than 100%, it will be equivalent to a fraction or a decimal less than 1. If a percent is more than 100%, it will be equivalent to fraction or decimal that is greater than 1.

Examples

1%	read as one percent	1 part out of 100
53%	read as fifty-three percent	53 parts out of 100
$17\frac{1}{2}$ %	read as seventeen and a half percent	$17\frac{1}{2}$ parts out of 100
$\frac{1}{2}$ %	read as one half percent	$\frac{1}{2}$ of one part out of 100
136%	read as one hundred thirty six percent	136 parts out of 100

There are three equivalent ways of writing the same number. One involves the use of fractions, one uses decimals and one uses percents.

One can say,

„I got $\frac{1}{2}$ of the questions right in my test."

„I got 0.5 of the questions right in my test."

„I got 50% of the questions right in my test."

All of these statements mean the same thing. The formal rules needed to change or convert from one format to another are as follows.

Percent to Decimals

Rules: 1. % can be replace by 0.01
2. Multiply by 0.01

Example 1

Change 42% to decimal

Solution: 42% = 42 x 0.01
= **0.42** Answer

The short way for this technique is to drop the % and shift the decimal point two places to the left, adding zeros if necessary.

Remember a percent that is less than 100% is equivalent to a decimal less than one.

If a percent is in fraction form, change it to decimal form before moving the decimal point.

Example 2

Change $\frac{3}{4}$ % to decimal.

Solution: $\frac{3}{4}$ % = 0.75% (Change $\frac{3}{4}$ to decimal)

= 0.75 x 0.01 (% replaced by 0.01)

= **0.0075** (multiply 0.75 by 0.01 or move decimal point two places to the left.

Example 3

Change 85% to decimal.

Solution: 85% = 85 x 0.01

= 0.85 Answer

Decimal to Percent

Rule: Multiply the decimal number by 100%.

Example 1

Change 0.57 to percent

Solution: 0.57 = 0.57 x 100%

= 57% Answer

The shortcut is to shift the decimal point two places to the right, add zeros if necessary, and insert the % sign.

Example 2

Change 3.7 to percent.

Solution: 3.7 = 3.7 x 100%

= **370%** Answer

Percent to Fraction

Rule: 1. Replace the % by $\frac{1}{100}$ and multiply.
2. Reduce to lowest term.

Example 1

Change 46% to fraction.

Solution: $46\% = 46 \times \frac{1}{100}$ (Replace % by $\frac{1}{100}$)

$= \frac{46}{100}$ (multiply)

$= \frac{\overset{23}{\cancel{46}}}{\underset{50}{\cancel{100}}}$ (Reduce to lowest term by dividing both 46 and 100 by GCF of 2)

$= \mathbf{\frac{23}{50}}$ **Answer**

If the percent has a decimal point in it, change it to a fraction before continuing.

Example 2

Change 1.2 % to fraction

Solution: $1.2\ \% = 1\frac{2}{10}\ \%$ (Change to a fraction)

$= \frac{12}{10} \times \frac{1}{100}$ (Replace % by $\frac{1}{100}$ and multiply)

$= \frac{12}{1000}$ (Reduce to lowest term)

$\mathbf{= \frac{3}{250}}$ Answer

Fraction to Percent

Rule: 1. Multiply by 100%
2. Change to a mixed number and reduce if necessary.

Example 1

Change $\frac{3}{4}$ to percent.

Solution: $\frac{3}{4} = \frac{3}{4} \times 100\%$

$= \frac{300}{4}\ \%$ Simplify the improper fraction

= 75% Answer

Example 2

Change $\frac{4}{5}$ to percent.

Solution: $\frac{4}{5} = \frac{4}{5} \times 100\%$

$= \frac{400}{5}\ \%$ Simplify the improper fraction

= 80% Answer

Some useful equivalents to remember are listed below.

Fraction	Decimal	Percent
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.50	50%
$\frac{3}{4}$	0.75	75%
$33\frac{1}{3}$	$0.\overline{3}\overline{3}\ldots$	$33\frac{1}{3}$ %
$66\frac{2}{3}$	$0.\overline{6}\overline{6}\ldots$	$66\frac{2}{3}$ %

REMEMBER:

- **To change percent to fraction, drop the % and shift or move the decimal point two places to the left, adding zeros if necessary.**

  If a percent is in a fraction form, change it to a decimal form before moving the decimal point.
- **To change decimal to percent, shift the decimal point two places to the right, add zeros if necessary and insert the % sign.**
- **To change percent to fraction, replace the % by $\frac{1}{100}$ and multiply reducing your answer to its lowest term.**
- **To change fraction to percent, multiply the fraction by 100%, change to mixed number and reduce the answer to its lowest term if necessary.**

NOW DO PRACTICE EXERCISE 13

Practice Exercise 13

1. Change each of the following percentages to an equivalent fraction in lowest term. The first one is done for you.

a) 20% $= 20 \times \frac{1}{100}$

$= \frac{\cancel{20}^{1}}{\cancel{100}_{5}}$

Answer: $= \mathbf{\frac{1}{5}}$

b) 40% =

Answer: __________

c) 70% =

Answer: __________

d) 35% =

Answer: __________

e) 4.5% =

Answer: ___________

f) 0.6% =

Answer: __________

2. Change each fraction to percentages. The first one is done for you.

a) $\frac{2}{5} = \frac{2}{5} \times 100\%$

$= \frac{200}{5}\%$

Answer: = 40%

b) $\frac{1}{8} =$

Answer: ___________

c) $\frac{7}{20}$ =

d) $\frac{9}{20}$ =

Answer: ____________

Answer: ____________

e) $\frac{3}{5}$ =

f) $\frac{7}{25}$ =

Answer: ____________

Answer: ____________

---

3. Convert each of the following percentages to decimals. The first one is done for you.

a) 15% = 15 x 0.01

Answer: **= 0.15**

b) 12% =

Answer: ____________

c) 36% =

Answer: ____________

d) 98% =

Answer: ____________

e) 68%

Answer: ____________

f) 450%

Answer: ____________

g) $\frac{2}{5}$ % =

Answer: ____________

h) $\frac{3}{8}$ % =

Answer: ____________

4. Change each of the following decimals to percentages. The first one is done for you.

a) 0.55 = 0.55 x 100%

Answer: **= 55%**

b) 1.50 =

Answer: ____________

c) 0.8

Answer: ____________

d) 0.66 =

Answer: ____________

e) 2.25 =

Answer: ____________

f) 0.07 =

Answer: ____________

---

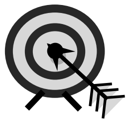
5. Complete the table.

Fraction	**Decimal**	**Percent**
a. $\frac{3}{8}$		
b.	0.52	
c.		4.5%
d. $\frac{1}{16}$		
e.	0.035	

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 14: Finding the Percentage of a Quantity

In Lesson 13, you learnt to convert percentages to fractions and decimals.



In this lesson, you will:

- find the percentage of a quantity
- solve problems by finding the percentage of a quantity in real life situations.

In the previous lesson, you learnt about the connections between percentages, fractions and decimals. This knowledge and skill will be needed as you go along in this lesson. Let us study some of the different types of problems that can be solved using percentages.

Finding the percentage of a quantity or an amount of money

Can you answer the following?

What is 25% of 16?

The question requires you to find a percentage of a certain quantity or number. That is, you are asked to find 25% of 16 or $\frac{1}{4}$ of 16. From this we can form the equation **25% of 16 = n** or $\mathbf{\frac{1}{4}}$ **of 16 = n**, where **n** refers to that certain part of 16.

Solving the equation, we have

Method 1 25% of 16 = n (Here, „**of**" indicates multiplication)

= 25% x 16

= 0.25 x 16 (Change 25% to decimal and multiply)

= 4 **Answer**

Method 2

25% of 16 = $\frac{1}{4}$ of 16 (change 25% = $\frac{25}{100} = \frac{1}{4}$)

= $\frac{1}{\cancel{4}} \times \cancel{16}$

= 4

Therefore, 25% of 16 is 4.

Note: To do arithmetic in percents, the percent must be changed to either its decimal or fractional equivalent. It is easier to do with decimals, and some will be easier to do with fractions. This is one skill you need to learn. That is**, deciding which is easier for you.**

> **REMEMBER:**
>
> **To find and work out a percentage of a quantity, write the percentage as a decimal or a fraction and multiply by the quantity.**

Example 1

What is 6.5 % of 200?

Solution: 6.5% of 200 = 6.5% x 200

= 0.065 x 200 (change 6.5% to decimal)

= 13

Therefore, 6.5% of 200 is 13.

Example 2

$10\frac{1}{5}$ % of 500 is what number?

Solution: $10\frac{1}{5}$ % of 500 = $10\frac{1}{5}$ % x 500

= 10.2% x 500 (Change$10\frac{1}{5}$ to decimal.)

= 0.102 x 500 (Multiply 10.2 by 0.01)

= 51

Therefore, $10\frac{1}{5}$ % of 500 is 51.

Example 3

Calculate 17% of 1000 m.

Solution: 17% of 1000 m = 17% x 1000 m

= 0.17 x 1000 m

= 170 m

Therefore, 17 % of 1000 m is 170 m.

Example 4

Work out 40% of 85 min.

Solution: 40% of 85 min = 40% x 85 min

= 0.40 x 85 min

= 34 min

Therefore, 40% of 85min is 34 min.

The knowledge of percentages allows us to solve many everyday problems easily. Let us see how and where it applies.

Example 1

In a basketball game, Ben scored 45% of the total score for his team. If the team scored 120 points, how many points did Ben make?

Solution: We reword the problem. That is, 45% of 120 points is how many?

In symbol, we write 45% of 120 = n

On the basis of the above example, we can now solve for n.

$45\% \text{ of } 120 = 45\% \times 120$

$= 0.45 \times 120$

$= 54$

Therefore, Ben scored 54 points.

Example 2

On a 20-item practice test, how many questions must Diana answer to receive a score of 80%?

Solution: Here you are asked to find 80% of 20.

So, you can form the equation 80% of 20 = n.

Solving for n, we have

$80\% \text{ of } 20 = n$

$n = 80\% \times 20$

$n = 0.80 \times 20$

$n = 16$

Therefore, Diana must answer 16 questions to get a score of 80%.

Example 3

Sonia saves 25% of her pay. How much money will she save from a pay of K82.60?

Solution: Let n represents Sonia"s savings.

Since Sonia saves 25% of her pay, then n = 25% of K82.60

Solving for n, we have n = 25% of K82.60

= 0.25 x K82.60

= K20.65

Therefore, Sonia saves K20.65.

Example 4

Mrs. Jacob bought a dress worth K120. She was given a discount of 5%; How much did she pay for the dress?

Solution: The advertised price of an article is the **list or regular price.** The reduction in this price is called the **discount**. A discount is usually stated as a percentage of the list price. The price after a discount has been deducted is called the **sale price**.

In the problem, the list price is K120. The rate of discount is 5%. We are looking for the sale price. First we find the discount.

To find the discount, we get 5% of K120.

So, Discount = 5% of K120

= 5% x K120

= 0.05 x K120

Sale price = List Price - Discount

= K120 – K6

= K114

Therefore, Mrs. Jacob paid K114 for the dress.

Example 5

Miro bought a household appliance which sells for K3500. He was charged 10% VAT. How much did he pay for the appliance?

Solution: **VAT (Value Added Tax)** is the tax added to the regular or sale price of goods that we buy from restaurants, stores or supermarkets. It is usually expressed as a percentage of the selling price.

In the problem, the selling price is K3500. We are looking for the buying price.

First, we find the Amount of VAT.

VAT = 10% of K3500

= 10% x K3500

= 0.10 x K3500

= K350

Buying Price = selling price + VAT

= K3500 + K350

= K3850

Therefore, Miro paid K3850 for the appliance.

Suppose you read in the newspaper that „The inflation for August is 8.5%"". What does this mean?

Inflation, here, means that **the prices of the goods are increasing**. For the month mentioned the rate is 8.5%. This implies that an item that cost K10 at the beginning of August probably cost K10.85 at the end of August. We say probably because prices of items do not increase at the same rate.

Example 6

What would you expect to pay, at the end of August, for a pencil case that cost K16 at the beginning of August if the rate of inflation is 8.5%?

Solution: To solve the problem,

a) Find the increase in price
b) Getting directly the inflated price.

a) Increase = 8.5% of K16

= 8.5% x K16

= 0.085 x K16

= K1.36

b) Inflated price = Original price + Increase in price

= K16 + K1.36

= K17.36

Therefore, the expected price at the end of August is K17.36.

NOW DO PRACTICE EXERCISE 14

Practice Exercise 14

1. Find the percentage of each of the following quantities as indicated. The first one is done for you.

a) 20% of 90 = 20% x 90
= 0.20 x 90
Answer: = 18

b) 15% of 250
Answer: __________

c) 40.5% of 400
Answer: __________

d) 30% of 660
Answer: __________

e) 36% of 175
Answer: __________

f) 12% of 350
Answer: __________

2. Calculate the value of each of the following.

a) 15% of 50 km
Answer: __________

b) 125% of K70
Answer: __________

c) 5% of 12 cm
Answer: __________

d) 95% of 30 Litres
Answer: __________

e) 62.5% of 400 kg
Answer: __________

3. Determine the sale price for each of the following items.

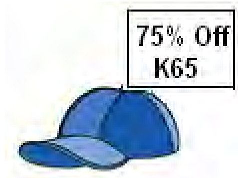
a)

b)

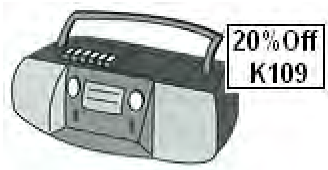
c)

Answer: __________ **Answer:**__________ **Answer:**__________

d)



e)



Answer:__________ **Answer:**__________

---

4. Larry received 5% commission as a salesman at Datec for selling a printing machine that cost K14 000. How much commission did Larry receive?

Answer:__________

---

5. What is the selling price of a guitar marked at K180 if it is advertised for sale at a $33\frac{1}{3}$ % discount?

Answer:__________

6. Jane bought a blouse marked at K98.50. She was charged 15% VAT.

   How much did she pay for the blouse? Write your answer to the nearest whole kina.

Answer:__________

---

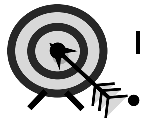
7. If a government employee is earning K2500 and will receive an increase of 15%, what will be his new salary?

Answer:__________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 15: Finding What Percentage of One Number is Another

In Lesson 14, you learnt how to work out the percentage of a quantity.



In this lesson, you will:

- find a number as a percentage of another
- solve problems by finding what percentage of one quantity is another quantity in real life situations.

Another type of percentage problem we are now going to look at is finding what percent one number is of another.

It is often useful to know the percentage that one quantity is of another.

Let us look at the examples.

Example 1

In a test of 50 questions, Tom made six mistakes. What percent of all questions were his mistakes?

Solution: The question means the same as „what percent of 50 is 6? If 6 is n% of 50, then, translating this into an equation, we have

$$n\% \text{ of } 50 = 6.$$

Now we solve the equation to find n.

$$n\% \text{ of } 50 = 6$$

$$n\% \times 50 = 6$$

$$n\% = \frac{6}{50} \qquad \text{(Divide by 50 on both sides.)}$$

$$n\% = 0.12$$

$$n = 0.12 \times 100\%$$

$$n = 12\%$$

Therefore, Tim's mistakes were 12% of the whole test.

Example 2

During a sale, Joseph was allowed to buy a cassette recorder marked at K1150 for K989.

How much was the discount? What rate percent was it?

Solution: a) Discount = K1150 – K989

= K 161

We are asked to find what percent of K1150 is K161. If n% is the rate of discount, then, to get the discount, we would have,

n% of K1150 = K161

n% x k1150 = K161

$$n\% = \frac{161}{1150}$$

n% = 0.14

n = 0.14 x 100%

n = 14%

Therefore, Joseph was allowed a discount of 14%.

The shortcut to solve these types of problems is outlined in the rule below.

To express one quantity as a percentage of another, write the first quantity or number as a fraction of the second and then convert to a percentage by multiplying by 100 (do not forget the % sign). Always ensure that the quantities are in the same units.

Now look at the following examples.

Example 1

Find 75 as a percentage of 300.

Solution: 75 out of 300 = $\frac{75}{300}$ x 100

= 0.25 x100

= 25%

Therefore, 75 is 25% of 300.

Example 2

What percentage of 100 g is 20 g?

Solution: 20 g out of 100 g = $\frac{20\text{ g}}{100\text{ g}}$ x 100

= 0.2 x100

= 20%

Therefore, 20 g is 20% of 100 g.

Example 3

90 min is what percent of 12 hours?

Solution: First, the two quantities should be expressed in the same units

so, $\frac{90\text{ min}}{12\text{ hours}} = \frac{90\text{ min}}{720\text{ min}}$ x 100 (1 hour = 60 min, so, 12 x 60 = 720)

= 0.125 x 100

= 12.5% or $12\frac{1}{2}$ %

Therefore, 90 min is 12.5% or $12\frac{1}{2}$ % of 12 hours.

Now, apply the technique you learnt into real life problems.

Percentages are used when people discuss issues involving changes in number. For example, retailers use percentages when discounting the price of goods for sale.

Example 4

A certain book was marked K75 but bought during a sale for K60. What was the percent discount?

Solution: First, we have to find the discount.

Discount = Marked price – Sale price

= K75 – K60

= K15

To find the percent discount, divide the discount by the marked price and multiply by 100.

Percent Discount = $\frac{15}{75}$ x 100

= 0.2 x 100

= 20 %

Therefore, the percent discount is 20%.

Example 4

The cost price of a jug is K69. The selling price is K46. What is the percentage discount?

Solution: Discount = K69 – K46

= K23

Percentage discount = $\frac{23}{69}$ x 100%

= 33.3 % or $33\frac{1}{3}$ **%**

Example 5

A school canteen sells 90 pies per day during winter and 30 pies during summer. What is the percentage decrease?

Solution: Decrease = 90 – 30

= 60

Percentage Decrease = $\frac{60}{90}$ x 100%

= $66.\bar{6}$ %

= $66\frac{2}{3}$ %

NOW DO PRACTICE EXERCISE 15

Practice Exercise 15

1. Express the first quantity as a percentage of the second in each of the following pairs of numbers.

a. 13 of 50 b. 77 of 100 c. 15 of 20

Answer: _________ **Answer:** _________ **Answer:** _________

d. 21 of 25 e. 16 of 40 f. 32 of 80

Answer: _________ **Answer:** _________ **Answer:** _________

---

2. Find the first as a percentage of the second. Express your answer as a whole number or as a decimal number correct to 2 decimal places.

a) 600 m of 3 km b) 40 m of 2 km c) 3 mm of 5 cm

Answer: _________ **Answer:** _________ **Answer:** _________

d) 25^t of K4 e) 66^t of K2 f) 30 sec of 4 min

Answer: _________ **Answer:** _________ **Answer:** _________

g) 750 g of 2.5 kg h) 7 days of 4 weeks i) 85 mL of 2 L

Answer: _________ **Answer:** _________ **Answer:** _________

3. Mr. West, the school principal, interviewed 40 of the students who left last year. Only 7 did not have a job.

   a) Find the percentage of students who did not have a job.

   Answer: __________

   b) Find the percentage of students who did have a job.

   Answer: __________

---

4. A 420 g tin of corn kernels contains sweet corn, water, salt and 4.2 g of added sugar.

   What percentage of the contents is added sugar?

   Answer: __________

---

5. Mr. and Mrs. Go decided to sell their house for K750 000. They paid the real estate agent K26 250 to sell the house.

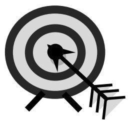
   What percentage is this of the selling price?

   Answer: __________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 16: Finding the Number When a Percentage of the Number is known

In Lesson 15, you learnt how to find a quantity as a percentage of another quantity.



In this lesson, you will:

- find a number when a percent of the number is known
- solve problems by finding a number when a percent of the number is known.

In the Problem „**8 is what percent of 20?'** we are asked to find a number as a percentage of another. The solution is found by writing the first quantity or number as a fraction of the second. Then convert to a percentage by multiplying by 100%.

But how would we solve this problem: **18 is 40% of what number?**

There are two ways of solving this problem.

Method 1

> **To find the number or quantity if a percentage of it is known, first find 1% or 10% then multiply by 100 to find 100% (the whole quantity).**

Let us consider the second problem above. **18 is 40% of what number?**

„18 is 40% of what number?" is the same as „40% of what number is 18?"

We write, 40% of the number = 18

Identifying 1% of the number = 18 ÷ 40

= 0.45

Therefore: 100% of the number = 0.45 x 100

= 45

Therefore, the number whose 40% is 18 is 45.

Method 2

> **To find the number or quantity if a percentage of the number is known, translate the problem into an equation relating factors to their product and convert the percent to decimals and divide.**

Since we do not know what the number is, we replace this with **n.**

We write, 40% of n = 18 or 40% x n = 18 or 0.40 x n = 18

By simply dividing 18 by 0.4 n = 18 ÷ 0.4

We get n = 45

Now let us look at the following worked examples.

Example 1

Joan spent 8% of her money on a new dress which costs K108. How much money did she have before the purchase?

Solution: **Using Method 1**

8% of Joan‟s money = K108

1% of Joan‟s money = K108 † 8

= K13.50

100% of Joan‟s money = K13.50 x 100

= K1350

Therefore, Joan’s money before the purchase is K1350.

Using Method 2

Since we do not know how much money Joan had before the purchase we write,

8% of n = K108

8% x n = K108

0.08 x n = K108

n = K108 ÷ 0.08

n = K1350

Example 2

In a one day cricket match, a batsman scored 102 runs. If this was 34% of the team‟s runs, what was the team‟s score?

Solution: **Using Method 1**

34% of the team‟s runs = 102

1% of the team‟s runs = 102 ÷ 34

= 3

100% of team‟s score = 3 x 100

= 300

Therefore, the team’s score is 300 runs.

Using Method 2

34% of n = 102

34% x n = 102

0.34 x n = 102

n = 102 ÷ 0.34

n = 300

Example 3

In a school, 25% of the teachers teach mathematics. If there are 40 mathematics teachers, how many teachers are there in the school?

Solution: **Method 1**

25% of the teachers = 40
1% of the teachers = 40 ÷ 25
= 1.6
100% of teacher"s = 1.6 x 100
= 160

Therefore, the total number of teachers in the school is 160 .

Using Method 2

25% of n = 40
25% x n = 40
0.25 x n = 40
n = 40 ÷ 0.25
n = 160

NOW DO PRACTICE EXERCISE 16

Practice Exercise 16

1. Using the method of your choice determine the size of a quantity if:

a) 94% of it is K564

Answer:__________

b) 30% of it is 27kg

Answer:__________

c) 28% of it is K29.12

Answer:__________

d) 80% of it is 2800

Answer:__________

e) 120% of it is 840

Answer:__________

f) 4% of it is 20 m

Answer:__________

g) 50% of it is 36

Answer:__________

h) 75% of it is 60

Answer:__________

2. Problem Solving

a) Ashley spent 12% of his money on a surfboard which cost K468. How much money did he have before the purchase?

Answer: ___________

b) In a driving school, 56 students passed their test for the first time. If this is 70% of the people who took their test for the first time, how many people took their test?

Answer: ___________

c) There were 23 200 spectators who came to a stadium to watch a concert. If this is 58% of the seating capacity of the stadium, what is the total seating capacity of the stadium?

Answer: ___________

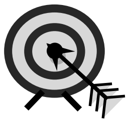
d) Jessica gave 60 roses to the church. If this is $66\frac{2}{3}\%$ of the roses she collected from her garden, how many roses did she collect from her garden?

Answer: ___________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 17: Simple Interest

In Lessons 14 to16, you learnt to work out different types of percentage problems.



In this lesson, you will:

- define simple interest
- identify the three factors which determines simple interest
- calculate the simple interest paid for a loan
- calculate the principal, the rate of interest and the length of time of the investment
- determine the total amount a person earns/pays at the end of the given period of a loan.

In Grade 7 Strand 1 Sub-strand 3 Lesson 19, you learnt the meaning of interest.

Interest is the amount of extra money paid in return for having the use of someone else's money.

When you borrow or save money with a bank, building society, credit union or a finance company, interest is **charged** or **paid**.

Interest is **charged** on money you borrow and **paid** on money you save.

Interest is charged or paid *per annum (pa)* and charged or paid at a fixed rate e.g. 6% pa. Per annum is a term that means each year.

Example 1

You borrow K350 for one year at 9%. How much do you pay back in total?

At the end of the year you will pay back: 100% of the K350 + 9% interest.

You will pay back a total of 109% of K350.

Calculating 109% of K350: 350 x 1.09 = 381.5

You will pay back a total of **K381.50**.

Example 2

You save K120 for one year at 4%. How much in total will you have?

At the end of a year you will have: 100% of your K120 + 4% interest

You will have a total of 104% of your K120.

Calculating 104% of K120: 120 x 1.04 = 124.8

You will have a total of **K124.80**.

Now let us look at simple interest.

Simple interest is a type of an interest which is fixed to the sum of money actually borrowed or saved.

When you borrow a sum of money, simple interest is charged for each year on the actual sum borrowed.

When you save money, simple interest is paid for each year on the actual money saved.

The amount of interest added to a loan, or added to a sum of money that is saved can be calculated using the formula:

$$\boxed{I = P \times R \times T}$$

This is known as the simple interest formula.

In words, the formula is:

Interest = Principal x Rate x Time

- The **Principal is the amount of money you borrow, lend or save.**
- The **Rate is the interest rate per annum expressed as a decimal.**
- The **Time is the period or for how long the money is borrowed. It is usually expressed in years.**

Simple Interest is calculated as a percentage of the principal (amount of money borrowed) per annum (pa) or per year.

Example 1

Calculate the interest charged or paid on a loan of K750, for 4 years at 7% per annum.

Solution: Use the formula $I = P \times R \times T$

Substitute P = K750, R = 7% or 0.07 and T = 4 years

So, $I = K750 \times 0.07 \times 4$

$I = K210$

The interest charged or paid on this loan is K210.

If the K750 had been saved for 4 years at 7% per annum, **the interest paid on the savings would have been K210.**

Example 2

To buy a new drum set, a band borrows K3500 over 10 years at 17% interest per annum.

a) Calculate the amount of interest paid on the loan.

b) At the end of the loan, how much in total will have been paid for the drum set?

Solution: a) Calculating the interest paid on the loan

Use the formula $I = P \times R \times T$

Substitute P = K3500, R = 17% or 0.17 and T = 10 years

So, $I = K3500 \times 0.17 \times 10$

$I = K5950$

The interest charged or paid on this loan is K5950.

b) Total Amount paid at the end of the loan = principal + interest

Total Amount = P + I
= K3500 + K5950
= K9450

At the end of the loan, a total of K9450 will have been paid for the drum set.

The formula for simple interest can be given in different forms.

To find the principal when the amount of interest, the time and the rate are given, we use the formula:

$$\boxed{P = \frac{I}{R \times T}}$$

To find the rate when the amount of interest, the time and the principal are given, we use the formula:

$$\boxed{R = \frac{I}{P \times T}}$$

To find the time when the amount of interest, the rate and the principal are given, we use the formula:

$$\boxed{T = \frac{I}{P \times R}}$$

Now look at the examples.

Example 3

Westin borrowed K4500 from the bank to start a business. The bank asked him to repay K5040 after one year. What was the rate of interest charged?

Solution: What is given: Amount borrowed (P) = K4500
Repayment (T.A.) = K5040
Time = 1 year

What is asked: Interest (I) = __________
Rate of interest (R) = __________

To find the Interest, subtract the amount borrowed (P) from the repayment (T.A.)

Interest = T.A. – P
= K5040 – K4500
= K540

To find the rate of interest, use the formula: $R = \frac{I}{P \times T}$

By substitution, $R = \frac{540}{4500 \times 1}$

$R = 0.12$ or 12%

Therefore, the rate of interest charged by the bank to Westin is 12%.

Example 4

To start his store, Lama borrowed an amount of money from a Savings and Loan Society at an interest rate of 10% per annum. After 3 years, he paid an interest of K360. How much did Lama borrow?

Solution: What is given: Interest (I) = K360
Time = 3 year
Rate of interest (R) = 10% or 0.1

What is asked: Principal (I) = __________

To calculate the principal, use the formula: $P = \frac{I}{R \times T}$

By substitution: $P = \frac{K360}{0.1 \times 3}$

= K1200

Therefore, Lama borrowed the amount of K1200.

Example 5

James invested K6000 in a bank that gives 8% interest per annum. How long will he need to invest the money in the bank to get K2400 interest?

Solution: What is given: Principal (P) = K6000
Interest (I) = K2400
Rate of interest (R) = 8% or 0.08

What is asked: Time = __________

To calculate the Time, use the formula: $T = \frac{I}{P \times R}$

By substitution: $T = \frac{K2400}{0.08 \times K6000}$

$= 5$ years

Therefore, James should invest his money for 5 years to get K2400 interest.

NOW DO PRACTICE EXERCISE 17

Practice Exercise 17

1. Calculate the simple interest charged or paid for each of the following:

a) K450 borrowed for 6 years with interest at 12% p.a.

Answer: __________

b) K280 saved for 8 years with a rate of interest of 3% p.a.

Answer: __________

c) K1500 saved for 15 years at 6% p.a. interest

Answer: __________

d) K6000 borrowed for 10 years at an interest rate of 17% p.a.

Answer: __________

e) K25 borrowed for 20 years with interest at 18% p.a.

Answer: __________

2. To buy a new car, Mr. Kua borrows K15000 over 5 years at 15% interest per annum.

a) Calculate the amount of interest paid on the loan.

Answer: __________

b) At the end of the loan, how much in total will he pay for the car?

Answer: __________

3. Thomas borrowed K3500 from the bank to start his poultry business. The bank asked him to repay K4480 after 2 years.

   What was the rate of interest charged?

Answer:__________

---

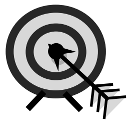
4. James invested K15 000 in a bank that gives 12% interest per annum. How long will he need to invest the money in the bank to get K7200 interest?

Answer:__________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 18: Compound Interest

In Lesson 17, you learnt the meaning of the words such as interest, rate of interest, per annum, simple interest and principal. You also learnt to calculate the simple interest and the amount for a deposit or a loan.



In this lesson, you will:

- define compound interest
- determine interest compounded annually, semi-annually and quarterly.

In a loan or savings transactions, interest arrangements may be simple or compounded. If you withdraw only the interest each year, then the interest is called simple interest. The money invested (principal) and the annual interest will be the same each year.

For compound interest, the interest for a year must be left in the savings account and added to that year‟s principal. So each year the money in the account and the annual interest both increase or accumulate.

- **The process of accumulating a principal to obtain a compound amount is called *compound accumulation*.**
- ***Compound interest* is the difference between the original principal and the *compound amount* (principal plus total interest).**
- ***Conversion period* is the period for computing interest, usually at regularly stated intervals such as annually, semi-annually, quarterly or monthly.**
- **The stated interest rate is called the *nominal rate.***

For example

If the nominal rate of interest is 8% compounded annually, the interest rate per conversion period is 8%.

If the nominal rate of interest is 8% compounded semi-annually, the interest per conversion period is $\frac{8\%}{2}$ or 4% (2 six-months per year)

If the nominal rate of interest is 8% compounded quarterly, the interest rate per conversion period is $\frac{8\%}{4}$ or 2 (4 three-months per year)

Compound interest can be calculated step by step. Work out the interest for a year then add the interest to the principal for that year. This gives you the principal for the next year. Repeat these steps for each year the money is invested.

To find the total interest, work out

Total Interest = Final Amount – First Principal

Example 1

Find the compound interest on K100 at 10% for three years and the total amount due at the end of three years.

Solution: Principal for the first year = K100
Interest for the first year = 10% of k100 = K10

Principal for the second year = K110
Interest for the 2^{nd} year = 10% of k110 = K11

Principal for the third year = K121
Interest for the 3^{rd} year = 10% of k121 = K12.10

Total amount due at the end of three years = K133.10
The compound interest is K133.10 – K100 = K33.10

Example 2

What is the total amount due after three years if you borrow K3000 at 10% p.a. compounded annually?

Solution: Principal for the first year = K3000
Interest for the first year = 10% of K3000 = K300

Principal for the second year = K3300
Interest for the 2^{nd} year = 10% of K3300 = K330

Principal for the third year = K3630
Interest for the 3^{rd} year = 10% of K3630 = K363

So, **Total amount due at the end of three years = K3993**

The compound interest is K3993 – K3000 = K993

Example 3

Charlie borrowed K2000 payable in 3 years at 6% compounded semi-annually. How much must he pay back at the end of the period?

Solution: 6% compounded semi-annually means that the interest per year is 6% and the interest is paid every six months or semi-annually.

Therefore, the interest rate per conversion period is $\frac{6\%}{2}$ or 3%.

To work out how much Charlie must pay back at the end of the period, we have

Principal for the first year = K2000
Interest for the first year = 3% of K2000 = K 60

Principal for the second year = K2060
Interest for the 2^{nd} year = 3% of K2060 = K 61.80

Principal for the third year = K2121.80
Interest for the 3^{rd} year = 3% of K2121.80 = K 63.65

Total amount due at the end of three years = K 2185.45 (to the nearest toea)

So, Charlie must pay back K2185.45 at the end of three years.

Example 4

Billy borrowed K1000 payable in 4 years at 12% compounded quarterly. How much must he pay back at the end of the period?

Solution: 12% compounded quarterly means that the interest per year is 12% and the interest is paid every three (3) months or quarterly.

Therefore, the interest rate per conversion period is $\frac{12\%}{4}$ or 3%.

To work out how much Charlie must pay back at the end of the period, we have

Principal for the first year = K1000
Interest for the first year = 3% of K1000 = K 30

Principal for the second year = K1030
Interest for the 2^{nd} year = 3% of K1030 = K 30.90

Principal for the third year = K1060.90
Interest for the 3^{rd} year = 3% of K1060.90 = K 31.827

Principal for the fourth year = K1092.727

Interest for the 3^{rd} year = 3% of K1092.727 = K 32.78181

Total amount due at the end of four years = K1125.50 (to the nearest toea)

So, Billy must pay back K2185.45 at the end of three years.

NOW DO PRACTICE EXERCISE 18

Practice Exercise 18

1. Calculate the compound interest charged or paid for each of the following:

a) K450 borrowed for 2 years with interest at 12% p.a. compounded annually

Answer: __________

b) K280 saved for 3 years with an interest rate of 3% p.a. compounded annually

Answer: __________

c) K1500 saved for 3 years at 6% p.a. interest compounded semi-annually

Answer: __________

d) K6000 borrowed for 4 years at an interest rate of 10% p.a. compounded annually

Answer: __________

e) K25 borrowed for 2 years with interest at 16% p.a. compounded quarterly.

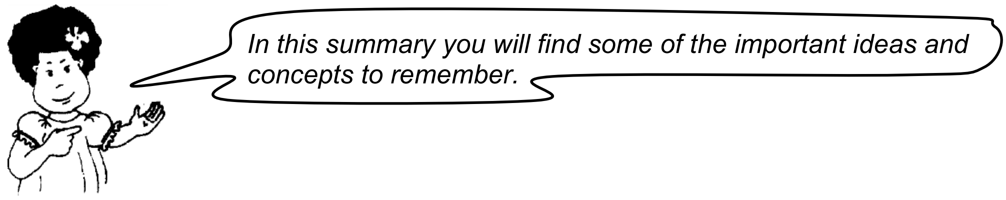
Answer: __________

2. Joe deposited K12 000 in a bank which pays 12% interest compounded quarterly. How much is the balance of Joe‟s account in the bank at the end of 3 years? Write your answer to the nearest ten toea.

Answer: __________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3

SUB-STRAND 3: SUMMARY



- **Percent** means out of a hundred or per hundred. The symbol for percent is %.
- There are three equivalent ways of writing the same number. One involves the use of fractions, one uses decimals and one uses percentages. All of these mean the same thing. The formal rules needed to change from one format to another are as follows:

  Percent $\longrightarrow$ Decimal

  -- Drop the % sign and shift the decimal point two places to the left, adding zeros if necessary.

  Decimal $\longrightarrow$ Percentage

  -- Shift the decimal point two places to the right, add zeros if necessary, and insert the % sign.

  Percentage $\longrightarrow$ Fraction

  -- Replace % by $\frac{1}{100}$ and multiply reducing your answer to its lowest term.

  Fraction $\longrightarrow$ Percentage

  -- multiply the fraction by 100%, change to mixed number and reduce the answer to its lowest term if necessary.
- **Interest** is the amount of extra money paid in return for having the use of someone else‟s money.
- **Simple interest** is an interest which is fixed to the sum of money actually borrowed or saved.
- **Principal** is the amount of money you borrow, lend or save.
- **Rate** is the interest rate per annum expressed in decimal or percentage**.**
- **Time** refers to the period of time the money is borrowed or invested. It is usually expressed in years.
- The process of accumulating a principal to obtain a compound amount is called ***compound accumulation***.
- ***Compound interest*** is the difference between the original principal and the *compound amount* (principal plus total interest).
- ***Conversion period*** is the period for computing interest, usually at regularly stated intervals such as annually, semi-annually, quarterly or monthly.
- The stated interest rate is called the ***nominal rate****.*

REVISE LESSONS 13 – 18. THEN DO SUB-STRAND TEST 3 IN ASSIGNMENT 1.

ANSWERS TO PRACTICE EXERCISES 13 - 18.

Practice Exercise 13

1. b) $\frac{2}{5}$ c) $\frac{7}{10}$ d) $\frac{7}{20}$ e) $4\frac{1}{2}$ f) $\frac{3}{500}$

2. b) 12.5% or $12\frac{1}{2}$ % c) 35% d) 45%
   e) 60% f) 28%

3. b) 0.12 c) 0.36 d) 0.98 e) 0.68
   f) 4.5 g) 0.004 h) 0. 00375

4. b) 150% c) 80% d) 66% e) 225% f) 7%

5.

Fraction	Decimal	Percent
a) $\frac{3}{8}$	**0.375**	**37.5% or $37\frac{1}{2}$ %**
b) $\mathbf{\frac{13}{25}}$	0.52	**52%**
c) $\mathbf{\frac{9}{200}}$	**0.045**	4.5%
d) $\frac{1}{16}$	0.0625	**6.25% or $6\frac{1}{4}$ %**
e) $\mathbf{\frac{7}{200}}$	0.035	**3.5% or $3\frac{1}{2}$ %**

Practice Exercise 14

1. b) 37.5 c) 162 d) 198 e) 63 f) 42
2. a) 7.5 km b) K87.50 c) 6 cm d) 28.5 L e) 250 kg
3. a) K123.25 b) 28.50 c) 11.70 d) 16.25 e) 87.2
4. K700
5. K120
6. K83.725 or K84
7. K2875

Practice Exercise 15

1.	a)	26%	d)	84%
	b)	77%	e)	40%
	c)	75%	f)	40%
2.	a)	20%	f)	12.5% or $6\frac{1}{2}$%
	b)	2%	g)	30%
	c)	6%	h)	25%
	d)	6.25% or $6\frac{1}{4}$%	i)	42.5% or $42\frac{1}{2}$%
	e)	33%		
3.	a)	17.5% or $17\frac{1}{2}$%		
	b)	82.5% or $82\frac{1}{2}$%		

4. 1%
5. 3.5% or $3\frac{1}{2}$%

Practice Exercise 16

1.	a)	K600	e)	700
	b)	90 kg	f)	500 m
	c)	K104	g)	72
	d)	3500	h)	80

2. a) K3900
   b) 80 people
   c) 40 000 seats
   d) 90 roses

Practice Exercise 17

1. a) K324
   b) K67.20
   c) K1350
   d) K10 200
   e) K90

2. a) K11 250
   b) K26 250

3. 14%
4. 4 years

Practice Exercise 18

1. a) K114.48
   b) K305.96
   c) K139.09
   d) K2 784.60
   e) K51.30

2. K13 112.70

END OF SUB-STRAND 3

SUB-STRAND 4

RATIO AND RATES

SUB-STRAND 4: RATIO AND RATES

Introduction

This sub-strand contains reviews of the meanings and uses of ratios and rates as well as the rules of working with ratios, rates and proportions that you learnt in Grade 7.

Special ratios and proportions have been used in architecture since ancient times. For example, The Golden Rectangle has a ratio of length is to width of approximately 1:1.62 and is considered very pleasing to look at. The Great Parthenon (built 447-432 BC), the temple on the Acropolis of Athens, is built entirely of marble, the Doric Temples, one of the largest as well as one of the finest measuring 70 m by 31 m. All these are evidences of the application of ratios and proportion.

Examples of ratios and proportions are found in many real life situations. One such situation occurs when you have to compare the parts of the human body to determine if they are in proportion. Also, when you have to add water to a juice concentrate to make up a fruit drink, when comparing the lengths of the top of your head to your waist and your waist to the bottom of your feet or when you divide or share an amount of money equally and so on.

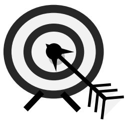
The picture below shows application of ratios and proportions in real life.

In this Sub-strand, you will:

- recognize and relate rates to graphs
- apply ratios and rates in solving problems in real life situations.

Lesson 19: Simplifying Ratios

You learnt about decimals, fractions and percentages in the previous lessons.



In this lesson, you will:

- compare two or more quantities
- express ratios in simplest form.

In Grade 7, you learnt something about ratio. Now you will extend more knowledge about ratio.

Ratio is a way of comparing one quantity to another. It is the answer obtained when two quantities are compared by division. It expresses a relation in size between these quantities.

We use ratio to compare sizes of sets or groups of quantities.

Example

The picture below shows 12 motorbikes and 10 cars displayed for sale. There are 12 motorbikes to every 10 cars. We can say that the ratio of the number of motorbikes to the number of cars is 12 is to 10.

Mathematically, the ratio 12 is to 10 can also be written as 12:10. The expression is read as „12 is to 10". 12 and 10 are called the **terms** of the ratio.

There are also other ratios that can be formed when we compare the numbers of motorbikes and cars.

Comparing Part to Whole

What is the ratio of the number of motor bikes to the total number of displayed vehicles on sale?

Solution: $$\frac{\text{12 motorbikes}}{\text{12 motorbikes +10 cars}} = \frac{\text{12 motorbikes}}{\text{22 vehicles}} = \frac{12}{22}$$

We say, the ratio is $\frac{12}{22}$ or 12 : 22.

Comparing Whole to Part

What is the ratio of the vehicles displayed on sale to the number of cars?

Solution: $\dfrac{\text{12 motorbikes + 10 cars}}{\text{10 cars}} = \dfrac{\text{22 vehicles}}{\text{10 cars}} = \dfrac{22}{10}$

We say the ratio is $\dfrac{22}{10}$ or 22 : 10.

Example 2

There are 24 girls and 12 boys in the school children‟s choir.
Find the ratio of:

a) girls is to boys?
b) boys is to girls?
c) girls is to children in the choir?
d) children is to boys?

Solution:

a) girls is to boys = 24 is to 12 = $\dfrac{24}{12}$ or 24 : 12

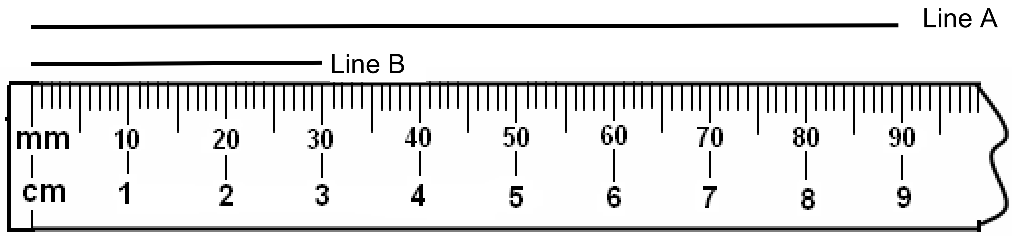
b) boys is to girls = 12 is to 24 = $\dfrac{12}{24}$ or 12 : 24

c) girls is to children = $\dfrac{24}{\text{12 boys + 24 girls}} = \dfrac{24}{12 + 24} = \dfrac{24}{36}$ or 24 : 36

d) children is to boys = $\dfrac{\text{24 girls + 12 boys}}{\text{12 boys}} = \dfrac{24+ 12}{12} = \dfrac{36}{12}$ or 36 : 12

We use ratios to compare quantities of the same kind. The numbers in a ratio should also be in the same units.

Example 1



The diagram indicates two lengths, the first by Line **A**, the second by Line **B**.

Using units of measurements, we obtained the table below.

Unit	**1mm**	**1cm**	**3cm**
Number of units in Line A	90	9	3
Number of units in Line B	30	3	1
Ratio	90:30	9:3	3:1

We may compare the first length to the second length by any of the equivalent ratios 90:30, 9:3, 3:1.

Example 2

Thus, when we compare the two lengths 12 cm and 4 cm, we may write either

$$12 \text{ cm} : 4 \text{ cm} = 12 : 4 \text{ or } \frac{12 \text{ cm}}{4 \text{ cm}} \text{ or } \frac{12}{4}$$

Because the units are the same, we can leave them out. If the units are different they should be changed to the same units before the ratio is simplified.

Simplifying Ratios

Ratios are simplified in a similar way as in fractions.

Example 1

Simplify 600 mL : 750 mL

Solution: 600 mL : 750 mL = 600 : 750 or $\frac{600}{750}$ (Divide both terms by 150)

= 4 : 5 or $\frac{4}{5}$ (Simplified form)

Example 2

Simplify the ratio 600 m : 2 km.

Solution: Change the numbers to the same unit

600 m : 2 km = 600 m : 2000 m or $\frac{600 \text{ m}}{2000 \text{ m}}$ (since 1 km = 1000 m)

= 600 : 2000 or $\frac{600}{2000}$

Simplify by dividing both terms by the 200 (GCF of 600 and 2000)

= 3 : 10 or $\frac{3}{10}$ (simplest form)

When there are fractions in a ratio, multiply both terms by a suitable common denominator.

Example 3

Simplify $\frac{1}{2} : \frac{1}{5}$

Solution: Multiply both terms by 10. (10 is the LCD of 5 and 2)

$\frac{1}{2} \times 10 : \frac{1}{5} \times 10 = 2 : 5$

NOW DO PRACTICE EXERCISE 19

Practice Exercise 19

1) Complete these ratio statements:

a) A class has 18 boys and 13 girls.

Boys : girls = __________ : __________

b) A drink mix has 2 parts cordial and 5 parts water.

Cordial : water = __________ : __________

c) A car park has 6 spaces for handicapped drivers and 75 other spaces.

Handicapped : others = __________ : __________

d) For every 5 dogs a veterinary clinic sees, they see 8 cats.

Cats : dogs = __________ : __________

e) A box contains 7 red balls and 12 yellow balls.

Red balls : yellow balls = __________ : __________

2. A school rugby squad was made up recently by players from the following team: Blue team, 9, Red team, 8 and green team, 6.

Write down the following ratios:

a) Blue team to red team

Answer: __________

b) Green team to others

Answer: __________

c) Red team to the whole squad

Answer: __________

3. Write the ratios in their simplest form.

a) 42 : 21

b) 4 : 10

c) 18 : 24

d) 65 : 195

e) $\frac{1}{2} : \frac{1}{3}$

f) 8 hours : 12 hours

g) 6 m : 16 m

h) 250 g : 350 g

i) K45 : K30

j) $\frac{3}{4} : \frac{5}{8}$

4. Express the ratio of the first quantity to the second quantity as simply as possible.

a) 2 hours : 40 minutes

b) 5 L : 250 mL

c) 2 m : 250 cm

d) 5 cm : 40 mm

e) 600 g : 3 kg

f) 500 m : 1 km

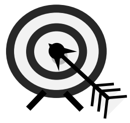
g) K35 : 150 toea

h) 75 toea : K2

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

Lesson 20: Using Ratios to Find Quantities

In Lesson 19, you learnt how to compare two or more quantities and express ratios in their simplest form.



In this lesson, you will:

- use the language of proportion to denote equal ratios.
- identify the names used for numbers in equal ratios.
- determine whether or not a proportion is a true statement.
- solve problem involving proportions.

In Grade 7, you learnt that equal ratios form a **proportion**. You also learnt that in any equal ratios, four (4) numbers are involved, namely the **means** and the **extremes**.

Look at the equal ratios below.

Extremes

$$1 : 2 = 3 : 6$$

Means

> **REMEMBER:**
>
> **The *extremes* are the end numbers in a proportion, while *means* are the middle numbers in a proportion.**

In the proportion above, 1 and 6 are the extremes and 2 and 3 are the means. Notice that the product of extremes is equal to the product of the means.

$$1 \times 6 = 2 \times 3$$
$$6 = 6$$

The proportion 1 : 2 = 3 : 6 can also be written in fraction form $\frac{1}{2} : \frac{3}{6}$. To check, whether $\frac{1}{2}$ and $\frac{3}{6}$ are equal or are in proportion, we apply the **Proportion Rule** or the process of **cross multiplication.**

$$\frac{1}{2} \quad \frac{3}{6}$$

$$1 \times 6 = 2 \times 3$$
$$6 = 6$$

Compare the products of the extremes and means to the products in cross multiplication. Which numbers in the fraction formed correspond to the extremes and to the means?

> **REMEMBER:**
>
> **The product of the means equals the product of the extremes.**

The equality of ratios can be checked through both methods. These are important in finding missing numbers in a proportion.

The following examples will show you how to find the missing number in an expression of equal ratios.

Example 1

Find the missing term in the statement $\frac{8}{10} = \frac{12}{n}$.

Solution: $\frac{8}{10} = \frac{12}{n}$

By cross multiplication: $8 \times n = 10 \times 12$

$8 \times n = 120$

$n = \frac{120}{8}$

$\mathbf{n = 15}$ **Answer**

Therefore, $\frac{8}{10} = \frac{12}{15}$.

Example 2

Find the value of n in the equality ratios 15 : 45 = n : 9.

Solution: 15 : 45 = n : 9

Multiply the means and the extremes,

$45 \times n = 15 \times 9$

$45 \times n = 135$

$n = \frac{135}{45}$

$\mathbf{n = 3}$ **Answer**

Therefore, 15 : 45 = 3 : 9.

To check, write the ratios as fractions.

$\frac{15}{45} = \frac{3}{9}$

$15 \times 9 = 45 \times 3$

$135 = 135$

Example 3 is similar to examples 1 and 2. In Example 4, the steps are simplified. Find out if you can trace the steps.

Example 3

Simplify $\frac{n}{7} = \frac{18}{21}$.

Solution: $\frac{n}{7} = \frac{18}{21}$

By cross multiplication,

$$n \times 21 = 7 \times 18$$
$$n \times 21 = 126$$
$$n = \frac{126}{21}$$

n = 6 **Answer**

Therefore, $\frac{6}{7} = \frac{18}{21}$.

To check: $\frac{6}{7} = \frac{18}{21}$

$$6 \times 21 = 7 \times 18$$
$$126 = 126$$

Example 4

What is the missing term? $2 : 3 = 8 : n$

Solution: $2 : 3 = 8 : n$

$$2 \times n = 3 \times 8$$
$$n = \frac{3 \times 8}{2}$$
$$n = \frac{24}{2}$$

n = 12 **Answer**

Therefore, $2 : 3 = 8 : 12$

To check: $2 : 3 = 8 : n$

$$2 : 3 = 8 : 12$$
$$2 \times 12 = 3 \times 8$$
$$24 = 24$$

The steps for solving a proportion may be summarized as follows:

Step 1 Set the product of the means equal to the product of the extremes or vice versa.

Step 2 Divide both terms of the resulting equation by the coefficient of the variable or letter (unknown or missing number).

Step 3 Check the result. Replace the unknown with the value obtained in the original proportion then cross multiply to verify that the proportion is true.

A proportion in which one of the four terms is missing or unknown is used to solve applied problems.

Look at the following examples.

Example 5

The ratio of Darius money to his sister‟s money is 1 is to 4. If Darius‟ money is K100, how much will his sister have?

Solution: Represent the sister‟s money by n.

Then, the ratios are 1 : 4 and 100 : n.

Let us take the ratio 1: 4 = 100 : n.

Solving for the value of n, we use the proportion rule, the product of the extremes equals the product of the means.

$$1 : 4 = 100 : n$$
$$1 \times n = 4 \times 100$$
$$\mathbf{n = 400}$$

To check: $1 : 4 = 100 : 400$

$$1 \times 400 = 4 \times 100$$
$$400 = 400$$

Therefore, Darius' sister will have K400.

Example 6

If Anne can type 60 words in one minute, how many words can she type in 20 minutes?

Solution: Translate the problem into a proportion

$$60 : 1 = n : 20$$
$$1 \times n = 60 \times 20$$
$$n = 1200$$

Therefore, Anne can type 1200 words in 20 minutes.

Example 7

Andrew draws a floor plan for his family‟s house. On his plan, 1 cm represents 4 m of the real house. What should be the length on his plan if the house is to be 80 metres long?

Solution: First ratio: 1 cm : 4 m

Second ratio: n cm : 80 m

Form the proportion by equating the two ratios.

$$1 \text{ cm} : 4 \text{ m} = n \text{ cm} : 80 \text{ m}$$

$$1 : 4 = n : 80$$

$$4 \times n = 1 \times 80$$

$$4n = 80$$

$$n = \frac{80}{4}$$

$$n = 20$$

Therefore, the length on his plan is 20 cm.

Note: In solving problems using proportion, you can use either the proportion rule or the cross multiplication method. Choose the one which you find easy.

NOW DO PRACTICE EXERCISE 20

Practice Exercise 20

---

1. Which of the following are proportions?

a) $2 : 9 = 4 : 18$

b) $1 : 2 = 2 : 1$

c) $\frac{1}{2} : \frac{1}{2} = 1 : 1$

d) $5 : 4 = \frac{1}{2} : \frac{2}{5}$

e) $60 : 100 = 6 : 1$

---

2. Find the value of **n**.

a) $2 : 3 = n : 21$

Answer:__________

b) $5 : 9 = 35 : n$

Answer:__________

c) $45 : n = 10 : 14$

Answer:__________

d) $\frac{n}{32} = \frac{24}{8}$

Answer:__________

e) $\frac{169}{n} = \frac{13}{15}$

Answer:__________

3. Work out the following word problems using proportion.

   a) Ten men can make a small house in 12 days. How long will it take 15 men to finish the same job?

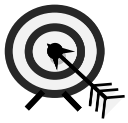
   b) The ratio of the earnings of a plumber and his helper is 3 : 2. If the helper received K120 as his share, how much did the plumber receive?

   c) A packet of fruit and nut mix is made up so that the ratio of the weight of dried fruit to the weight of nuts is 3:7. If there is 150 g of dried fruit, calculate the weight of nuts.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

Lesson 21: Changing Quantities in a Given Ratio

In Lesson 20, you learnt to use the language of proportion to denote equal ratios and identify the terms involved in equal ratios. You also determined whether or not a proportion is a true statement, solved a proportion for an unknown term and use proportion to solve applied problems.



In this lesson, you will:

- describe increase or decrease of quantities in a given ratio
- calculate the increase or decrease of quantities using ratios.

Changes in numbers such as an increase in the population of a city or town, or a decrease in the amount of sales and an increase or decrease in the salary of government employees can be described using ratios.

When we say changing a quantity in a given ratio we mean that we increase or decrease a number or an amount in a certain ratio. The ratio always compares the change to the old or original quantity.

- **If the ratio of a new quantity to an old quantity can be expressed as an improper fraction, then the new quantity is greater than the old quantity. Applying this ratio to the old quantity is known as increasing the old quantity in a given ratio.**
- **If the ratio of a new quantity to an old quantity can be expressed as a proper fraction, then the new quantity is less than the old quantity. Applying this ratio to the old quantity is known as decreasing the old quantity in a given ratio.**

Example 1 Increase 20 in the ratio 3 : 2.

Solution: An increase in the ratio 3 : 2 implies that

New quantity : Old quantity = 3 : 2

If we let **n** as the new quantity, then n : 20 = 3 : 2

Using the Proportion Rule or Cross multiplication;

$2 \times n = 20 \times 3$ or $\frac{n}{20} = \frac{3}{2}$

$2n = 60$ $2n = 60$

$n = 30$ $n = 30$

Therefore, the new quantity is **30.**

Example 2 Increase 60 kilograms in the ratio 19 : 15.

Solution: An increase in the ratio 19 : 15 implies that,

New Quantity : Old quantity = 19 : 15

If we let **n** as the new quantity, then n : 60 = 19 : 15

Using the Proportion Rule or Cross multiplication;

$$n \times 15 = 60 \times 19 \quad \text{or} \quad \frac{n}{60} = \frac{19}{15}$$

$$15n = 1140 \qquad 15n = 1140$$

$$n = 76 \qquad n = 76$$

Therefore, the new quantity is **76 kilograms.**

Example 3 Decrease 32 in the ratio 3 : 4.

Solution: A decrease in the ratio 3 : 4 implies that,

New Quantity : Old quantity = 3 : 4

If we let **n** as the new quantity, then n: 32 = 3 : 4

Using the Proportion Rule or Cross multiplication;

$$4 \times n = 32 \times 3 \quad \text{or} \quad \frac{n}{32} = \frac{3}{4}$$

$$4n = 96 \qquad 4n = 96$$

$$n = 24 \qquad n = 24$$

Therefore, the new quantity is **24.**

Example 4

Decrease 50 metres in the ratio 2 : 5.

Solution: A decrease in the ratio 2 : 5 implies that,

New Quantity : Old quantity = 2 : 5

If we let **n** as the new quantity, then n : 50 = 2 : 5

Using the Proportion Rule or Cross multiplication;

$$n \times 5 = 50 \times 2 \quad \text{or} \quad \frac{n}{50} = \frac{2}{5}$$

$$5n = 100 \quad \text{or} \quad 5n = 100$$

$$n = 20 \qquad n = 20$$

Therefore, the new quantity is **20 metres.**

A number or quantity may be increased or decreased by multiplying it by the given ratio expressed in fraction form.

Look at the following examples.

Example 5

An employee‟s salary is K12 000 per year when working 5 days a week. If the employee can only work 3 days per week, what will be the new salary?

Solution: New salary : Old salary = n : K12 000 = 3 : 5

$$n = K12000 \times \frac{3}{5}$$

$$n = K7200$$

Therefore the new salary is K7200.

Example 6

The school has 45 teachers. This number is to be increased in the ratio 11: 9 for next year. How many teachers will there be next year?

Solution: New staff : Old staff = 11: 9

$$n: 45 = 11: 9$$

$$n = 45 \times \frac{11}{9}$$

$$n = 55$$

Therefore, there will be 55 teachers next year.

Example 7

A property which was valued at K32 000 has increased in value in the ratio 47: 40. What is its value now?

Solution: New value : Old value = 47: 40

$$n: K32\ 000 = 47: 40$$

$$n = 32\ 000 \times \frac{47}{40}$$

$$n = 37\ 600$$

Therefore, the value of the property now is K37 600.

NOW DO PRACTICE EXERCISE 21

Practice Exercise 21

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1. Change the following quantities into the ratios as indicated.

   a) Increase 10 in the ratio 7: 2.

   Answer: ___________

   b) Decrease 39 in the ratio 12 : 13

   Answer: ___________

   c) Increase 165 in the ratio 9 : 5

   Answer: ___________

   d) Decrease 40 kg in the ratio 5 : 8

   Answer: ___________

   e) Decrease 90 litres in the ratio 7:10.

   Answer: ___________

   f) Increase 32 kilometres in the ratio 2 : 1

   Answer: ___________

---

2. Lucy had her weekly salary of K250 increased in the ratio 6:5. Find her new weekly salary.

   Answer: ___________

3. A newsagent orders 84 papers per day. Because of holidays, this is to be reduced in the ratio 2 : 3. How many papers will be ordered?

Answer: __________

---

4. A farmer grew 250 pawpaw trees in a year. The number of pawpaw trees this year is to increase in the ratio 7 : 5. How many pawpaw trees will there be this year?

Answer: __________

---

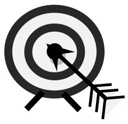
5. A factory plans to increase its sales in the ratio 3 : 1 next year. How many does it hope to sell next year if it sells 156 000 articles now?

Answer: __________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4

Lesson 22: Ratios and Rates

In the previous lesson, you calculated the increase and decrease of quantities using ratios.



In this lesson, you will:

- differentiate rates as distinct from ratios
- express measurement units in rates
- relate ratios to graphs.

As you know, ratio is a comparison of like quantities, or a comparison of two quantities of the same kind. As a result, they do not contain any units.

Example: 50mL of water to 25 mL of cordial. Both numbers are capacities measured in milliliters.

In a ratio, the measurement units are the same and they cancel each other.

The ratio can be simplified and written as: 50 mL : 25 mL = 2 : 1

A rate is a comparison of two quantities that have different units.

Example: 5 litres of water to 1 cup of detergent
60 km to 2 hours
6 small balls to 18 big balls
2 mugs of water to 4 teaspoon of coffee

As seen in the above examples, we say

Rate is a comparison of unlike quantities.

In a rate, the measurement units are different, so they do not cancel.

Note: To make rates easier to work with, we usually change the rate to its equivalent rate where we write down how many of the first quantity corresponds to one of the second quantity. Rates can be changed (as with ratios) by multiplying or dividing both quantities by the same number.

For example:

a) 150 km in 3 hours $= \frac{150}{3}$ km in $\frac{3}{3}$ hr = 50 km in 1 hr **= 50km/hr**

b) 32 baskets in 4 hours = $\frac{32}{4}$ baskets in $\frac{4}{4}$ hr = 8 baskets in 1 hr = **8 baskets/hr**

Unit rate is a simplified form of ratio between two measurements in which the second term is 1.

The most commonly used rates are:

1. **Speed** = which compares the distance traveled to time taken.

   Example: 60 km : 2 hours = 30 km : 1 hour = 30km/hr

2. **Exchange rate** = which compares one currency to another currency

   Example: The exchange rate between the New Zealand dollar and the Australian dollar is NZ$1 = A$0.80

> **A rate is a measure of how one quantity changes with respect to another.**

Now let us look at some basic rate problems.

The solutions of rate problems are also similar to the solutions of ratio problems.

Example 1

A woman travels 210 km in 3 hours. Express this as a rate in simplest form.

Solution: 210 km in 3 hours can be simplified by dividing each term by 3

so, 210 km in 3 hours $= \frac{210}{3}$ km in $\frac{3}{3}$ hr

= 70 km in 1 hr

= 70km/hr

Example 2

A mining company supplies ore at the rate of 1500 tonnes per day. How many tonnes can the mining company supply in 4 weeks?

Solution: a) 1500t/day

Since 4 weeks = 28 days

Rate in 4 weeks =1500t x 28t in 28 days

= 42 000t

Therefore, 42 000t can be supplied in 4 weeks.

Example 3

If rope costs K2.20 per metre, find the cost of 12 m of rope.

Solution: Since the cost per metre = K2.20

Cost of 12 metres of rope = K2.20 x 12

= K26.40

Therefore, 12 metres of rope cost K26.40.

Example 4

If an athlete ran 240 m in 30 seconds, what was her running rate or speed in metres per second?

Solution: 240 m in 30 seconds = $\frac{240}{30}$ m in $\frac{30}{30}$ sec

= 8 m in 1 sec

= 8m/sec

Therefore, the running rate is **8m/sec.**

REMEMBER:

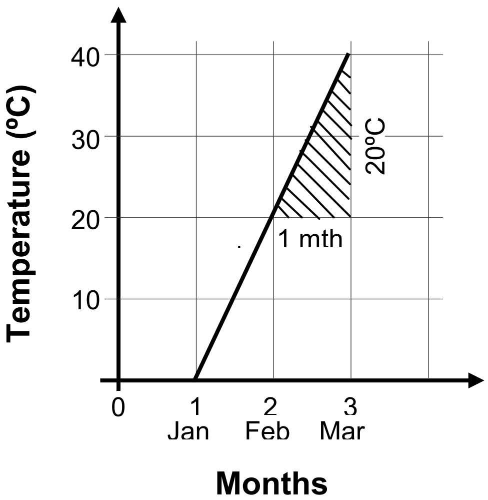
- **To simplify rates, multiply or divide both terms by the same number, just as you do with ratios.**
- **When using rates, you must show the units.**
- **Rates are usually expressed by writing down how many of the first quantity corresponds to one of the second.**

We can relate rates to graphs.

Consider the example below.

Example 1

In a town in the Northern Hemisphere, the temperature increases each month according to the graph below.



In two months, the average temperature increases from 0ºC to 40ºC.

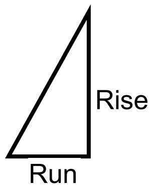
As a rate this is written as: $\frac{40°C}{2\,mth}$ = 20ºC per month

This rate can also be presented and shown on the graph as a triangle (as shown with sloping lines). This is called a **rate triangle**.

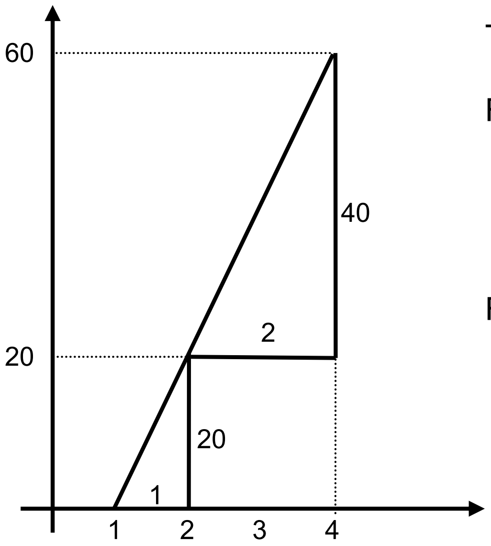
A rate triangle is a triangle that starts from a point on the graph line and goes right by some distance then turns up vertically until it meets the graph line again.

The rate triangle has a *run* and a *rise*.

The rate is calculated by the formula: Rate = $\frac{\textbf{Rise}}{\textbf{Run}}$



Look at the diagram below.



The diagram shows two rate triangles.

For the large triangle, the rate is calculated as:

$$\text{Rate} = \frac{\text{Rise}}{\text{Run}} = \frac{40°\text{C}}{2\text{mth}} = 20°\text{C/month}$$

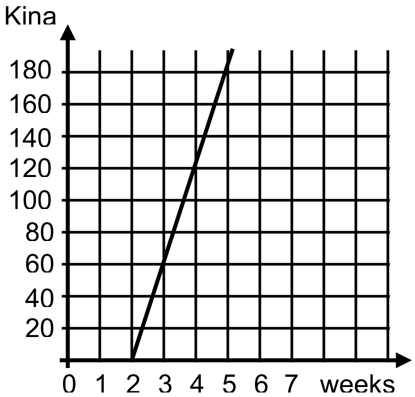
For the small triangle, the rate is calculated as:

$$\text{Rate} = \frac{\text{Rise}}{\text{Run}} = \frac{20°\text{C}}{1\text{mth}} = 20°\text{C/month}$$

This means that if the rate graph is a straight line, then it does not matter whether it is a small triangle or a large triangle, you get the same value for the rate.

Now let us study the graph below and work out the questions that follow.

PITA'S BANK BALANCE



According to the graph, Pita‟s bank balance increases.

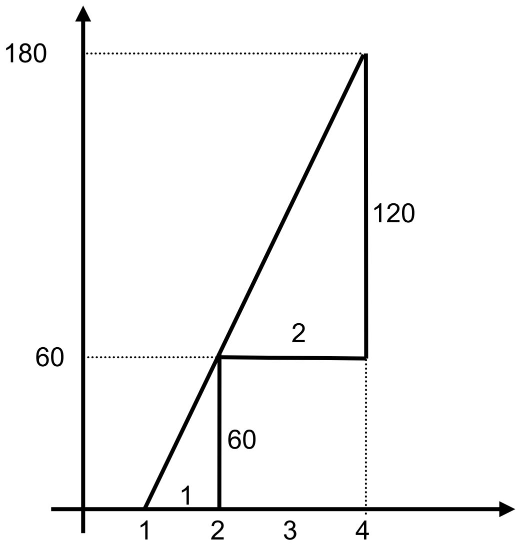
a) Calculate the rate of increase of the balance in kina per week.
b) Draw a rate triangle to show how your calculated answer obeys the rule: rate equals rise over run.

Solution:

a) In two weeks, the balance increases from 0 to K120.

As a rate, this is: $\frac{\text{K120}}{\text{2 wk}}$ = K60 per week

b) The rate triangle looks like this:



For the large triangle,

$$\text{Rate} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{K120}}{\text{2 wk}} = \text{K60/wk}$$

For the small triangle,

$$\text{Rate} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{K60}}{\text{1wk}} = \text{K60/wk}$$

NOW DO PRACTICE EXERCISE 22

Practice Exercise 22

1. Write each of the following as a rate in simplest form.

a) 8 km in 2 hours

Answer: __________

b) 5 km in 20 minutes

Answer: __________

c) K50 for 2 kg

Answer: __________

d) 60 chickens in 3 hours

Answer: __________

e) 120 children for 5 teachers

Answer: __________

2. Solve the following:

a) A jet plane is traveling at 600 km per hour. How far will it travel in 25 minutes?

Answer: __________

b) A car uses petrol at the rate of 9.5 litres per 100 km. How many litres of petrol would be used in traveling 350 km?

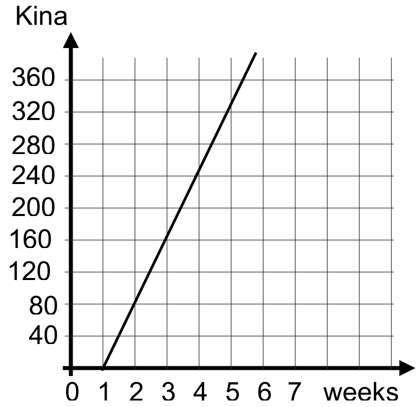
Answer: __________

c) Diana runs the marathon in 2 hours 25 minutes. If she covers 42.2 km, what is her average speed?

Answer: __________

3. **Refer to the diagram and answer the questions that follow.**

ROGER'S BANK BALANCE



a) Calculate the rate of increase of the balance in kina per week.

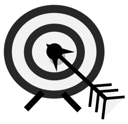
Answer: __________

b) Draw a rate triangle to show how the rate of increase is calculated.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

Lesson 23: Ratios with Three Terms

In Lesson 22, you learnt to distinguish rates from ratio and relate them to graphs.



In this lesson, you will:

- solve problems in real life situations involving ratios with three terms.

We use ratios in sharing or dividing quantities into proportional parts. The ratio tells us how to do the sharing or division.

Sharing something in a given ratio is called proportional division.

For example, if we divide K10 between two people giving K4 to one and K6 to the other. We consider the K10 as having 10 parts, of which 4 is given to one and 6 to the other. We say that the money is divided between the two people in the ratio 4: 6.

This is the same for example, if cement and sand are mixed in the ratio of 1 : 2, then 1 part of cement and 2 parts of sand equals 3 parts of concrete. We think of having 1 shovel of cement and 2 shovels of sand equals to three shovels of concrete or 1 bucket of cement and 2 buckets of sand equals three buckets of concrete.

Conversely, if we have a 45 cm length of wood and wish to divide it into three lengths which are in the ratio 2 : 3 : 4, we think of the wood as having 2 + 3 + 4 or 9 parts, each part being $\frac{45}{9}$ cm = 5 cm long. The three pieces will then have lengths of 10 cm (2 parts), 15 cm (3 parts) and 20 cm (4 parts) respectively.

To share, divide or split something (can be a number, an amount of money, a measurement, and so on) **in a given ratio, find:**

- **the total number of parts from the ratio**

  e.g. The ratio a: b : c gives a + b + c parts
- **what one part is**
- **the amounts for the ratio**

In this lesson, we will look at problems where the ratios are in three terms.

Study the examples on the preceding pages.

Example 1

Divide K840 between A, B and C in the ratio 5:7:9.

Solution: The ratio of the shares is 5 : 7 : 9

Consider K840 to be divided into 5 + 7 + 9 = 21 parts,

If we denote one part as x, then A = 5x, B =7x and C = 9x.

Therefore, 5x + 7x + 9x = K840

$$21x = 840$$

$$x = \frac{840}{21}$$

$$x = K40$$

To find the amount of each share, we have

A‟s share = 5x = 5(40) = **K200**

B‟s share = 7x = 7(40) = **K280**

C‟s share = 9x = 9(40) = **K360**

To check: We add the three shares:

k200 + K280 + K360 = K840

Example 2

A piece of land 720 square metres is divided into three smaller lots so that the area of the lots are in the ratio 2 : 3 : 4. Find the area of each lot.

Solution:

The ratio of the lots is 2 : 3 : 4

The whole piece of land is divided into 2 + 3 + 4 = 9 parts.

If we denote one part as x, then the area of the first piece of the lots = 2x, the second piece is 3x and the third piece is 4x.

Therefore, the area of one part 2x + 3x + 4x = 720

$$9x = 720$$

$$x = \frac{720}{9}$$

$$x = 80 \text{ m}^2$$

To find the area of each lot:

2x = 2(80) = **160 m²**

$2x = 2(80) = \mathbf{160\ m^2}$

$3x = 3(80) = \mathbf{240\ m^2}$

$4x = 4(80) = \mathbf{320\ m^2}$

To check: We add the three areas:

$160\ m^2 + 240\ m^2 + 320\ m^2 = 720\ m^2$

Example 3

Find three numbers whose sum is 2373 if the ratio of these numbers is 3 : 7 : 11.

Solution:

Consider the number 2373 to be divided into 3 + 7 + 11= 21 parts.

If we denote one part as x, then the first number is 3x, the second is 7x and the third is 11x.

To find x, we have $3x + 7x + 11x = 2373$

$$21x = 2373$$

$$x = \frac{2373}{21}$$

$$x = 113$$

To find the three numbers:

$3x = 3(113) = \mathbf{339}$ **(first number)**

$7x = 7(113) = \mathbf{791}$ **(second number)**

$11x = 11(113) = \mathbf{1243}$ **(third number)**

To check: We add the three numbers: $339 + 791 + 1243 = 2373$

Example 4

The sides of a triangle are in the ratio 5 : 6 : 7. If the distance around the triangle is 63 cm., find the length of each side.

Solution: Consider the length 63 cm to be divided into 5 + 6 + 7 = 18 parts.

If we denote one part as x, then the first number is 5x, the second is 6x and the third is 7x.

To find x, we have $5x + 6x + 7x = 63$

$$18x = 63$$

$$x = \frac{63}{18}$$

$$x = 3.5 \text{ cm}$$

To find the three numbers:

$5x = 5(3.5) = 17.5$ cm **(first side)**

$6x = 6(3.5) = 21$ cm **(second side)**

$7x = 7(3.5) = 24.5$ cm **(third side)**

To check: We add the three numbers: $17.5 + 21 + 24.5 = 63$

Example 5

Three employees, Mr. Ling, Miss Brown and Mrs. Kull hold 120, 200 and 40 shares respectively in their company. A total dividend of K1800 is paid to the three employees in the same ratio as their shares. How much does each employee receive?

Solution: The ratio of the shares is 120 : 200 : 40

In simplest form this is 3 : 5 : 1 (Divide by 40)

The total dividend (K1800) is divided into 3 + 5 + 1 = 9 parts

If we denote one part as x, then Mr. Ling‟s dividend = 3 x

Miss Brown‟s dividend = 5x

Mrs. Kull‟s dividend = x

To find x, we have 3x + 5x + x = K1800

$$9x = 1800$$

$$x = 200$$

Thus, Mr Ling‟s share = 3x = 3(200) = **K600** (Multiply 200 by 3)

Miss Brown‟s share = 5x = 5(200) = **K1000** Multiply 200 by 5)

Mrs. Kull‟s share = x = **K200**

To check, add the three shares: K600 + K1000 + K200 = K1800

NOW DO PRACTICE EXERCISE 23

Practice Exercise 23

1. Divide the following quantities in the ratios indicated:

   a) 156 km in the ratio 5 : 4 : 3.

   Answer: ___________

   b) 450 g in the ratio 5 : 7 : 13.

   Answer: ___________

   c) K70 in the ratio 2 : 3 : 5.

   Answer: ___________

   d) 80 cm in the ratio 4 : 7 : 9.

   Answer: ___________

---

2. Divide K980 between three girls Sofia, Sarah and Ruth in the ratio 2 : 5 : 7.

   Answer: ___________

---

3. Find three numbers whose sum is 450 and whose ratio is 2 : 3 : 5.

   Answer: ___________

4. The length, width and height of a room are in the ratio 4 : 3 : 2. If the sum of all the quantities is 1125 cm, find the dimensions of the room.

Answer: __________

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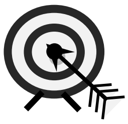
5. A piece of land 1350 square metres is divided into three smaller lots so that the area of the lots are in the ratio 3 : 5 : 7. Find the area of each lot.

Answer: __________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

Lesson 24: Using Ratios to Solve Problems

In Lesson 23, you learnt to solve and work out problems in real life situations involving ratio with three terms.



In this Lesson you will:
- use ratio and rates to solve simple problems.

We have a variety of problems in real life where the answers can be found by using ratios and proportion.

For example: Jacob‟s fish is four times longer than mine. How long is Jacob‟s fish if my fish is 30 cm long?

In the statement „Jacob‟s fish is four times longer than mine‟ does not give the actual size of each fish. However, once the length of one fish is known, the length of the other can be calculated.

In many problems, we are told the ratio of two quantities. We are then given one quantity and asked to calculate the other.

To solve word problems involving rates, ratios and proportion, apply the following steps.

Step 1 Read the problem carefully and identify the given information.
Step 2 Write the proportion needed to solve the problem. Represent the unknown quantity with a letter. Include the units in writing the proportion.
Step 3 Solve and check the proportion as before.

Example 1

Dress making cloth costs K6 per metre. How much does 5 metres cost?

Solution: Let the cost be x.

Ratio 1: K6 : 1 m

Ratio 2: Cost (x) : 5 m

Form a proportion by equating the two ratios

Ratio 1 = Ratio 2

$$\frac{K6}{1\text{ m}} = \frac{x}{5\text{ m}}$$

$$x(1\text{ m}) = K6(5\text{ m}) \quad \text{(Find the cross products)}$$

$$x = K30$$

Therefore, the cost of 5 metres of cloth is K30.

Example 2

The Science Club of a certain school sponsored a fund-raising project to construct the roof of the school‟s Botanical Garden. The club officers paid K1840 for 240 square metres of roofing materials. They still need to purchase 90 square metres more of the roofing material. How much would they have to pay for this additional material?

Solution: Let the amount they have to pay be x.

Ratio 1: K1840 : 240 sq. m

Ratio 2: K x : 90 sq.m

Form the proportion by equating the two ratios.

Ratio 1 = Ratio 2

$$\frac{\text{K1840}}{\text{240 sq.m}} = \frac{\text{K x}}{\text{90 sq.m}}$$

240 sq. m(Kx) = 90 sq. m (K1840) (Find the cross products)

$$x = \frac{165600}{240}$$

x = K690

Therefore, the club has to pay K690 more for the additional material.

Example 3

A car travels 55 km in 3 hours. How far will it travel in 10 hours if it continues at the same speed?

Solution: Ratio 1: 55 km : 3 hours

Ratio 2: x km : 10 hours

Form the proportion by equating the two ratios;

Ratio 1 = Ratio 2

$$\frac{\text{55 km}}{\text{3 hours}} = \frac{\text{x km}}{\text{10 hours}}$$

x km (3 hours) = 55 km(10 hours) (Find the cross products)

$$x = \frac{550}{3}$$

x = 183.33 km

Therefore, the car will travel 183.33 km in 10 hours.

Example 4

In a small town, 2 out of every 3 people are expected to have flu during winter. How many people would be expected to have flu, if the town has a population of 11 100 people?

Solution: Let x be the number of people expected to have flu.

Ratio 1: 2 : 3

Ratio 2: x : 11 100

Form the proportion by equating the two ratios;

Ratio 1 = Ratio 2

$$\frac{2}{3} = \frac{x}{11\,100}$$

$$3x = 2(11\,100) \qquad \text{(Find the cross products)}$$

$$x = \frac{22\,200}{3}$$

$$x = 7400 \text{ people}$$

Therefore, 7400 people are expected to have flu out of 11 100.

Example 5

Tom and Gary decided to divide the profits from a partnership business venture in the same ratio as their investments. If Tom invested K9000 and Gary invested K5000, how much should Gary receive if Tom received K3600?

Solution: Let Gary‟s share be x.

Ratio 1: K9000 : K3600
Ratio 2: K5000 : x

Form the proportion by equating the two ratios;

Ratio 1 = Ratio 2

$$\frac{K9000}{K3600} = \frac{K5000}{x}$$

$$9000x = 3600(5000) \qquad \text{(Find the cross products)}$$

$$x = \frac{18000000}{9000}$$

$$x = K2000$$

Therefore, Gary received K2000.

NOW DO PRACTICE EXERCISE 24

Practice Exercise 24

1. Solve for **x** in the following

a) x : 5 = 8 : 4

Answer: __________

b) x : 10 = 6 : 12

Answer: __________

c) $\frac{x}{19} = \frac{2}{38}$

Answer: __________

d) $\frac{x}{4} = \frac{7}{2}$

Answer: __________

2. Use ratio and proportion to solve the following problems.

a) The ratio of boys to girls in the library is 2 : 1. If there are 8 boys, how many are girls?

Answer: __________

b) Jack and Jill share some money in the ratio 5 : 3. If Jack receives K205, How much does Jill receive?

Answer: __________

c) Louisa can type 60 words in one minute. At this rate, how many words can she type in 20 minutes?

Answer: __________

d) Job and Ben started a business. Job invested an amount of K1 237 500 and Ben invested K450 000. They agreed to divide their profit of K225 000 in proportion to their investments.

How much did each receive?

Answer: __________

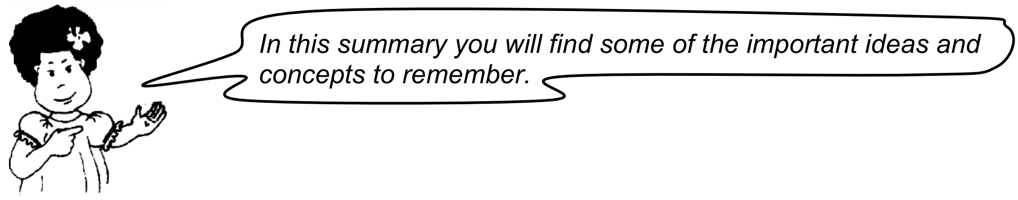
e) There are about 4 kilograms of muscle for every 11 kilograms of our body weight.

About how much of a person‟s weight is muscle if he weighs 70 kilograms?

Answer: __________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

SUB-STRAND 4: SUMMARY



- **Ratio** is a way of comparing one quantity to another. It is the answer obtained when two quantities are compared by division. It expresses a relation in size between these quantities.
- A statement that two ratios are equal is called a **proportion.**
- The ***extremes*** are the end numbers in a proportion, while the ***means*** are the middle numbers in a proportion.
- The product of the means is equal to the product of the extremes.
- **In solving a proportion,**
  - Set the product of the means equal to the product of the extremes or vice versa.
  - Divide both terms of the resulting equation by the coefficient of the variable or letter (unknown or missing number).
  - Check the result. Replace the unknown with the value obtained in the original proportion; then cross multiply to verify that the proportion is true.
- A **rate** is a measure of how one quantity changes with respect to another.
- **Unit rate** is a simplified form of ratio between two measurements in which the second term is 1.
- To simplify rates, multiply or divide both terms by the same number, just as you do with ratios.
- When using rates, you must show the units.
- **Rates** are usually expressed by writing down how many of the first quantity corresponds to one of the second.
- If the **ratio** of a new quantity to an old quantity can be expressed as an **improper fraction,** then the new quantity is greater than the old quantity. Applying this ratio to the old quantity is known as **increasing** the old quantity in a given ratio.
- If the **ratio** of a new quantity to an old quantity can be expressed as a **proper fraction**, then the new quantity is less than the old quantity. Applying this ratio to the old quantity is known as **decreasing** the old quantity in a given ratio.
- Sharing something in a given ratio is called **proportional division**.

REVISE LESSONS 13 – 18. THEN DO SUB-STRAND TEST 4 IN ASSIGNMENT 1.

ANSWERS TO PRACTICE EXERCISES 19 - 24

Practice Exercise 19

1. a) 18 : 13
   b) 2 : 5
   c) 6 : 75
   d) 8 : 5
   e) 7 : 12

2. a) 9 : 8 b) 6 : 17 c) 8 : 23

3.

a)	2 : 21	f)	2 : 3
b)	2 : 5	g)	3 : 8
c)	3 : 4	h)	5 : 7
d)	1 : 3	i)	3 : 2
e)	3 : 2	j)	6 : 5

4.

a)	3 : 1	e)	1 : 5
b)	20 : 1	f)	1 : 2
c)	4 : 5	g)	7 : 3
d)	5 : 4	h)	3 : 8

Practice Exercise 20

1. a, c and d

2. a) 14
   b) 63
   c) 63
   d) 96
   e) 195

3. a) 18 days
   b) K180
   c) 350 grams

Practice Exercise 21

1. a) 35 d) 25
   b) 36 e) 63
   c) 297 f) 64

2. K300

3. 56 papers

4. 350 pawpaws

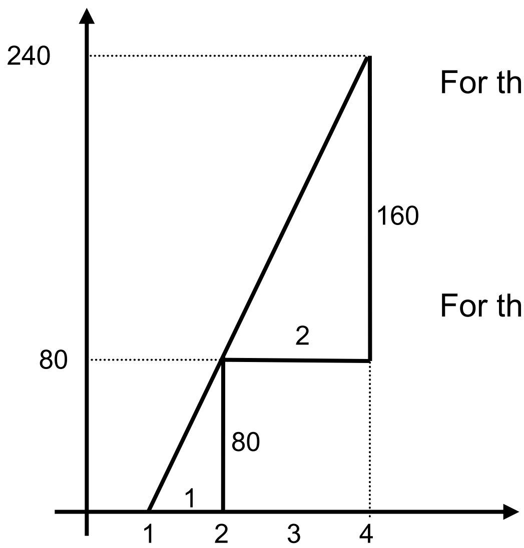
5. 468 000 articles

Practice Exercise 22

1. a) 4 km per hour
   b) $\frac{1}{4}$ km per min or 0.25 km per min
   c) K25 per kg
   d) 20 chickens per hour
   e) 24 children per teacher

2. a) 250 km
   b) 33.25 L
   c) 17.44 km per hour

3. a) K80
   b) Rate Triangle



For the large triangle,

$$\text{Rate} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{K160}}{\text{2 wk}} = \text{K80/wk}$$

For the small triangle,

$$\text{Rate} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{K80}}{\text{1wk}} = \text{K80/wk}$$

Practice Exercise 23

1. a) 65 mm, 52 mm, 39 mm
   b) 90 g, 126 g, 234 g
   c) K14, K21, K35
   d) 16 cm, 28 cm, 36 cm

2. Sofia, K140; Sarah, K350; Ruth, K490

3. 90, 135 and 225

4. L = 500 cm, W =375 and H = 250

5. 270 m^2, 450 m^2, and 630 m^2

Practice Exercise 24

1. a) 10
   b) 5
   c) 1
   d) 14

2. a) 4 girls
   b) K123
   c) 1200 words
   d) Job, K165 000 and Ben, K60 000
   e) 25 kg or 26 kg

END OF SUB-STRAND 4

NOW YOU MUST COMPLETE ASSIGNMENT 1.
RETURN IT TO THE PROVINCIAL COORDINATOR.

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